

DUAL-WATER ARCHIE, and the IMPORTANCE of WATER
GEOMETRY. A MODEL and a DISCUSSION.

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ABSTRACT

A dual-water Archie model for the interpretation of resistivity well logs is developed. From that development, a philosophy for Archie's parameters emerges that is quite different from some literature in our industry. In this paper, a model of electrical current behavior in the water in rock is presented. This model is based on the very fundamental electrical law relative to the conversion of resistance to resistivity, and the efficiency of the network of pores and pathways in the rock through which the electrical-survey current must flow. Electrical law, demonstration proofs and derivations are employed in order to promote a better understanding and a better use of Archie's relationships.

The geometry of water occupying void space in rock is shown to be the most important factor in controlling efficiency of the flow of electrical-survey current through interstitial-water paths in rock, thus giving value to Archie's parameters.

The classic Archie saturation equation (1942) emerges from the model presented herein, and in doing so is shown to be a dual-water dual-porosity relationship, and is extended to address shaly sands and additional levels of heterogeneity and practical reservoir complexity.

In addition, the paper illustrates how heterogeneities such as clay shale and semi-conductive minerals influence resistivity relationships. The model further illustrates both the resistivity behavior in the presence of hydrocarbon and the problems of interpretation in partially oil-wet and oil-wet rock.

The Archie parameters m and n serve special functions in electrical-current flow in both water-wet- and oil-wet rocks. An exhaustive exploration of saturation exponent n reveals behavior that has never before been explained in literature.

The two most important developments arising from this paper are the presentation of the model for the dual-water Archie method and the demonstration of the behavior of the saturation exponent n under in situ conditions. The demonstrated behavior of the water-saturation exponent n is considerably different from that behavior propounded by conventional wisdom.

Concurrently with the development of the Archie dual-water relationship from the model is the emergence of a second equation that employs a single exponent m_2 that replaces the two commonly used exponents m and n in the Archie saturation equation. The second equation opens an avenue to calculate water saturations without the knowledge of any part of m or n and precludes any requirements for their evaluation.

The reality that many oil- and gas- formations are complex in terms of mineralogy, lithology, wettability and saturation distributions makes a better understanding of the analytical process imperative.

AUTHOR'S NOTE TO THE READER: This is the latest version of the paper(05-10-2018)that has a much broadened presentation of the model and includes a very comprehensive study of the behavior of the saturation exponent n relative to both oil-wet and water-wet rocks.

WHAT ARE ARCHIE'S BASIC RELATIONSHIPS

Most water saturation equations used in resistivity well-log interpretation are based in some way on Archie's relationships. Resolve those equations to clean rocks, and Archie's basic water-saturation equation emerges. Archie's basic relationships (1942) most often are modified by location-specific empirical coefficients and exponents that are not compatible with basic electricity or rock properties. Whether or not any of these modifications provide solutions to local problems is of no interest in these discussions. The purpose of this paper is to dispel misunderstandings, prevent misuse, and promote better interpretations through better understanding. Archie's basic relationships are:

$$S_w^n = R_0 / R_{t \text{ measured}}$$

$$F = 1.0 / (\phi)^m$$

$$R_0 = FR_w$$

where, at the present time, the formation factor equation in many log analysis applications has been modified to

$$F_t = a / (\phi_t)^m$$

where the a coefficient most often is given a value lower than 1.0.

Each parameter in these equations will be developed graphically and mathematically, and through this development a conception of what each represents will emerge.

THE GRAPHICAL MODEL

Archie's *classic* relationships usually are considered to be clean-sand relationships. In shaly sands where clay shales produce additional electrical conductivity, Archie's relationships are said not to apply. Relative to Archie's original concept and its popular use, this pronouncement is correct. In the concept born of this model, an overall version of Archie's relationships emerges that shows that Archie's concept is a dual-water dual-porosity concept that also applies to shaly sands and other heterogeneous rocks that exhibit uniformity within the depth of investigation of the logging tools. This model is used to derive and define each of the terms in Archie's relationships, and the parameters derived and defined apply to any other methodology that makes use of Archie's parameters.

Figure 1 is an illustration of the fundamental resistivity model serving as the basis of this concept. This model first was

introduced in Ransom (1974), again in Ransom (1995), and will be shown in greatest detail herein. This figure is right-facing in left-to-right format. A **Figure 1** in reversed format, right-to-left, is furnished for the convenience of readers who are more familiar with left-facing diagrams. Figure 1 is not intended to be a working graphical procedure, it is extremely informative and explanatory at a basal level. This figure illustrates how bulk-volume water ($S_{wt}\phi_t$) is related to a unit volume of rock with resistivity (R_t). The model in Figure 1 is illustrative in nature and is not drawn to scale. The line drawing in the X-axis has been expanded so that detail can be observed and discussed. The figure is designed primarily to illustrate the electrical behavior of a volume of formation water as its environment changes with variations of insulating rock and insulating fluid.

In **Figure 1**, the origin of the diagram is represented by R_{we} (and $F_t = 1.0$) when ϕ_t is 1.0 or 100%. Where it commonly has been believed there are only two slopes in Archie's concept, the model demonstrates that there actually are three slopes, each representing exponents in Archie's concept. They are m_1 , m_2 , and n . The first is m_1 that pertains to the single parameter ϕ_t . This is the familiar porosity exponent m known in industry. Sometimes for the sake of clarity the term m_1 will be used instead of m . The second exponent is m_2 which pertains to two parameters, ϕ_t and S_{wt} , and is the exponent for the product $S_{wt}\phi_t$. The third is the exponent n that pertains only to S_{wt} , and is the saturation exponent commonly known in industry. Each of these exponents pertains to a resistivity gradient, the rate that resistivity changes as the volume of water in rock changes. The minimum resistivity gradient, or minimum value, for the saturation exponent n is the value of the porosity exponent m . A value for n that is lower than the value of m often is used by petrophysicists in their literature. This is contrary to physics. For the value of n to be lower than the value of m , the displacement of water by hydrocarbon must make the remaining water more electrically conductive, a violation of physics. It has never been explained in literature how the displacement of electrically-conductive water by hydrocarbon, under either in situ conditions or laboratory conditions, can increase the electrical conductivity of the remaining formation water.

From the diagram, the total fractional volume of water in the rock is represented by the projection on the X-axis under the two slopes, representing m and n (or m_2), drawn from 100% water at R_{we} to the intersection of the line representing specific slopes extrapolated to intercept resistivity level R_t . The fraction of water in total pore volume, S_{wt} , and the fraction of water in the total rock volume, $S_{wt}\phi_t$, are depicted on the X-axis by $\log\phi_t$ and $\log S_{wt}$. The fraction of total rock volume that is water provides the electrical conductivity to the rock resulting in resistivity R_t . As oil or gas displaces water, the water saturation S_{wt} decreases to the right (in the right facing **Figure 1**) as the

saturation of oil or gas ($1.0 - S_{wt}$), increases.

It is illustrated in the model in Figure 1 that there can be conditions related to the presence of hydrocarbon, oil in particular, that cause in situ rock resistivities to increase to extraordinarily high values. In oil-wet rocks the values of n will increase greatly (Keller, 1953; Sweeney and Jennings, 1960). The presence of oil in both water-wet and oil-wet rocks produces an increase in rock resistivity, but, in oil-wet rocks, the presence of oil causes very exaggerated interference to the flow of electrical-survey current. Resistivity R_t then will be increased correspondingly with the wettability to oil and its resulting electrical interference. The right-facing Figure 1 also shows that, under these conditions, when the commonly used default values of $n = m$ are employed, the line representing the slope of exponent n will be extended far to the right to intersect the level of the measured or derived value of R_t at a location H that would suggest a low value of S_{wt} . The lowest water saturations and the highest corresponding values of oil saturation, that can be calculated for the input data, occurs at point H. Values for saturation exponent n that are lower than the values of porosity exponent m are commonly seen in literature and are perpetuated by conventional wisdom. However, the employment of such values for n would cause the slope or resistivity gradient to decrease and the extended slope to intersect the R_t level beyond point H at artificially low water saturations. The saturation range for reasonable calculated water saturations is between irreducible water saturation and irreducible hydrocarbon saturations. How reasonable the estimated water saturation will be depends on the validity of exponent n or exponent m_2 .

This illustrates the resistivity interpretation problem in oil-bearing rocks where resistivity is exaggerated by the properties of oil. Here, to repeat, when the default value of n is lower than m , the extrapolated slope for n intersects the resistivity level R_t far to the right beyond point H in the model in the right-facing **Figure 1**, suggesting that water saturation is very low. When the common default value for n or any other unusually low value for n is employed, the derived saturations from the basic Archie equation, or from any more comprehensive equation, might not be correct and should be used with extreme caution in any field or reservoir description. This is a problematic but common occurrence when using the usual Archie-based resistivity interpretation methods by both private and commercial organizations. In the presence of oil, particularly in oil-wet rock where resistivity is high, the value of exponent n always is greater than the value of either exponent m_1 or m_2 .

An exploration of the graphic model will promote a better understanding of resistivity behavior in rocks. This exploration will be accompanied by a parallel algebraic development of the

terms and parameters in the basic Archie relationships. What each term or parameter represents is the key to understanding where to begin to solve interpretation problems.

This exploration will be concluded with a discussion of three unusual, but vintage, resistivity well logs, with special attention devoted to exponent n . Not only will exponent n be shown to be a resistivity gradient, but also is shown to be a measure of effectiveness of the electrical resistivity interference caused by the presence, distribution, and wettability to oil at high and low water saturation levels.

WHAT IS MEANT BY THE PLOT OF R_t VERSUS $S_{wt}\phi_t$

In **Figure 1**, the Y-axis is R_t , the resistivity of rock. The X-axis is $S_{wt}\phi_t$, bulk volume water. Bulk volume water is made up of two parts, ϕ_t and S_{wt} , which when added together on logarithmic scales, become $S_{wt}\phi_t$. On the X-axis, bulk volume water is dimensional and has resistivity. Kindly refer to the **PREFACE** at the beginning of the **APPENDIX** for further explanation.

In **Figure 1**, again, the origin of the diagram is represented by R_{we} at 100% porosity, but R_{we} is one of the products of Figure 2. **Figure 2** is a detailed view of an interior part of Figure 1 showing how R_w in the presence of dual water becomes R_{we} to become the origin of the diagram. The value R_{we} , as an equivalent water resistivity, is determined algebraically from Eq.(1b). The proportions of R_w and R_{wb} that become R_{we} in the figure are a function of water saturation as well as clay shale content. Within Figure 2, ϕ_e and ϕ_t exist simultaneously, and no matter what are their values, the same or different, when R_w becomes R_{we} , R_0 becomes R_0 corrected.

In **Figure 1**, R_t is seen plotted on a log-log plot versus the volume $S_{wt}\phi_t$, of which ϕ_t is a part. In resistivity well-log interpretation, the m or n slope in every case represents only the slope between two points: the value of the resistivity of the equivalent water, and the value of the resistivity of the total rock volume filled with the same water in whatever fraction and physical distribution. This fractional volume can be ϕ_t when $S_{wt} = 1.0$ and slope $m = m_1$, or it can be $S_{wt}\phi_t$ when $S_{wt} \leq 1.0$ and slope $m = m_2$. Slope n represents the case where the two resistivity end points for the slope are the resistivity of a rock completely filled with the equivalent water, and the resistivity of the same rock and the same water after hydrocarbon has displaced some of the water. There is no extrapolation and no interpolation involved in the m or n evaluation when the two end points are known. The resulting slope, be it m or n , is a measure of the difficulty and interference electrical-survey current experiences as it is forced to flow through the water in the rock and represents a resistivity gradient. In **Figure 1**, the line

representing the slope n rotates throughout the range of arc δ shown in the diagram as either or both n and S_{wt} vary. The steeper the slope for either m or n , the more inefficient will be the water path in the rock for conducting electrical-survey current, the greater will be the values of m and n , and the greater will be the value of R_t . Each value applies only to the individual sample of interest, whether in situ or in the laboratory.

The slopes representing values of m or n or m_2 are trigonometric tangential ratios of the side opposite (on Y-axis) over the side adjacent (on X-axis) of any right triangle in the figure. Exponents m and n and m_2 represent rates of change in resistivity R_t relative to changes in water volume ϕ_t or saturation S_{wt} , and reflect the efficiency or inefficiency of that water volume to conduct electrical-survey current. In the following examples, it will be shown how each exponent m , n and m_2 is derived as trigonometric tangents and calculated from their respective right triangles, and what can be derived from each.

It can be seen from the triangle CDG where n is the tangent that slope n is equal to

$$n = (\log R_t - \log R_{0 \text{ corrected}}) / (\log 1 - \log S_{wt})$$

$$-n(\log S_{wt}) = (\log R_t - \log R_{0 \text{ corrected}})$$

Therefore, $(S_{wt})^n = R_{0 \text{ corrected}} / R_t \quad . . . (4b)$

This is the derivation of Archie's dual-water equation that uses the two exponents, m and n .

Also, it can be seen in the triangle AEG where m_2 is the tangent

$$m_2 = \log F_t / (\log 1 - \log (S_{wt} \phi_t))$$

$$-m_2(\log (S_{wt} \phi_t)) = \log F_t$$

$$F_t = 1.0 / (S_{wt} \phi_t)^{m_2} \quad . . . (3b)$$

Please note the single exponent for the bulk volume of water. The formation factor, F_t , as it is derived here from Figure 1, applies to all values of $S_{wt} \leq 1.0$ and is represented on the resistivity axis as the difference between the logarithms of resistivity of the equivalent water (R_{we}) in the rock and the resistivity of the rock containing that water (R_t). This difference is expressed as $(\log R_t - \log R_{we})$, or R_t / R_{we} . So, when the equivalent R_t / R_{we} is substituted for F_t , we have

$$R_t / R_{we} = 1.0 / (S_{wt} \phi_t)^{m_2}$$

or
$$(S_{wt}\phi_t)^{m_2} = R_{we}/R_t$$

and further resolved becomes

$$S_{wt}^{m_2} = (1/\phi_t^{m_2}) (R_{we}/R_t) \quad . . . \quad (4d)$$

This is a water saturation equation derived from triangle AEG that uses a single exponent that is equivalent to Archie's equation in Eq.(4b) above that uses two exponents. The commonality of the exponents can be observed. Furthermore, it can be seen that one of the terms in the equation is R_{we} instead of R_0 corrected. Both R_{we} and R_0 corrected are dual-water dual-porosity derivatives and signify that the water saturation equations (4b) and(4d) are dual-water dual-porosity equations. Equation (4d) is an alternative equation to Archie's two-exponent equation, Eq. (4b). In Equation (4d), bulk volume water can be, and is, evaluated by core analysis, and sometimes under in situ conditions by downhole well-logging instruments. But it might not be necessary to evaluate $S_{wt}\phi_t$ as a combined term. Water saturation S_{wt} might be determined indirectly as seen in Eq.(4d) by using well logs recorded from resistivity- and porosity-measuring downhole instruments. Exponent m_2 is evaluated from triangle AEG, similarly to the measurement of exponent m or n . The value of m_2 will reside between the slopes representing values of exponents m and n . The fact that neither exponent m nor exponent n is required or used, in the evaluation of water saturation by the single-exponent method is no small matter.

Archie's saturation relationship is a straightforward derivation from the model, but is improved for use in dual-water dual-porosity methodology by the correction of R_w to R_{we} and R_0 to R_0 corrected. Both corrections can be observed in graphic form in **Figure 2**.

See the **APPENDIX (B)** for a more detailed explanation of Figures 1 and 2 and the derivations of m and n , and F_t and S_{wt} .

In addition, in an examination of the line slopes in Figure 1, it can be seen that $(S_{wt}\phi_t)^{m_2}$ has the same function as, and is equivalent to, $(S_{wt})^n (\phi_t)^{m_1}$. It is important to remember this in the developments that follow. For proof of this equality, see the **APPENDIX (B) (3)**.

SUMMARY OF EQUATIONS

TABLE 1	
EQUATION NUMBER	PURPOSE
Equation No. (1a) for R_{we} $1/R_{we} = (\phi_e/\phi_t)1/R_w + (\phi_{ne}/\phi_t)1/R_{wb}$	Equation for R_{we} for 100% water saturated environment. Involves two waters. Converts single-water method to dual-water method.
Equation No. (1b) for R_{we} $1/R_{we} = ((S_{we}\phi_e)/(S_{wt}\phi_t))1/R_w + (\phi_{ne}/(S_{wt}\phi_t))1/R_{wb}$	Equation for R_{we} for $\leq 100\%$ water saturated environment. Involves two waters. Converts single-water method to dual-water method.
Equation No. (1c) for a $a = R_{we}/R_w$	For the a coefficient. Shows the conversion from R_w to R_{we} .
Equation No. (2a) for R_0 $R_0 \text{ corrected} = F_t R_{we}$	For R_0 in 100% water saturated rock.
Equation No. (2b) for R_t $R_t \text{ calculated} = F_t R_{we}$	For $R_t \text{ calculated}$ in $\leq 100\%$ water saturated environment.
Equation No. (3a) for F_t $F_t = 1.0/(S_{wt}\phi_t)^{m_2} = 1.0/(\phi_t)^{m_1}$	For F_t conversion from oil-bearing environment to 100% water saturated environment.
Equation No. (3b) for F_t $F_t = 1.0/(S_{wt}\phi_t)^{m_2}$	For universal F_t in $\leq 100\%$ water saturated environment.
Equation No. (3c) $(S_{wt}\phi_t)^{m_2} = (S_{wt})^n (\phi_t)^{m_1}$	Exponent equivalence equation.
Equation No. (3d) $r \text{ ohms} = ((L/A)m/m^2) (R \text{ ohms } m^2/m)$	Converts resistance to resistivity and converse.
Equation No. (4a) for R_t $R_t \text{ measured} = (1.0/((S_{wt}\phi_t)^{m_2})) aR_w = (1.0/((S_{wt}\phi_t)^{m_2})) R_{we}$	Calculates R_t while using single-exponent m_2 saturation method.

Equation No. (4b) for S_{wt} $S_{wt}^n = (1.0 / (\phi_t^{m1})) (R_{we} / R_{t \text{ measured}})$	Dual-water Archie equation using two exponents, m and n , from triangle CDG of the model.
Equation No. (4c) for S_{wt} $S_{wt}^n = R_{0 \text{ corrected}} / R_t = (F_t R_{wz}) / (F_t R_{wa})$ $= R_{wz} / R_{wa}$	Water saturation equation from Figure 4, utilizing graphic determination of R_z .
Equation No. (4d) for S_{wt} $S_{wt}^{m2} = (1.0 / (\phi_t^{m2})) (R_{we} / R_{t \text{ measured}})$	Dual-water saturation equation using equivalent single exponent method from triangle AEG of the model.
Equation No. (5a) for m $m = (\log R_0 - \log R_w) / (\log 1 - \log \phi_e)$	Evaluation of m from raw data from triangle ABC of the model.
Equation No. (5b) for m $m = (\log R_{0 \text{ corrected}} - \log R_{we}) / (\log 1 - \log \phi_t)$	Evaluation of m from corrected data.
Equation No. (6) for S_{we} $S_{we} = 1.0 - (\phi_t / \phi_e) (1.0 - S_{wt})$	Calculation of water saturation in effective pore space.
Equation No. 4b) solved for exponent n $n = (\log R_t - \log R_0) / (\log 1 - \log S_{wt})$	Calculation of exponent n in both water-wet and oil-wet environments.
Equation No. 4d) solved for exponent m_2 $m_2 = (\log R_t - \log R_{we}) / (\log 1 - \log (S_{wt} \phi_t))$	Calculation of exponent m_2 in both water-wet and oil-wet environments.
End	End

PARALLEL RESISTIVITY EQUATIONS USED IN RESISTIVITY INTERPRETATIONS

The basic premise is that the host rock is an insulator. Equations will be demonstrated for the case of shaly sand with a uniform distribution of constituents and pore space. Shale is a term used to refer to fine grained, fissile, sedimentary rock. The term *shale* is a descriptive property of rock, not a mineral, therefore, shale may be referred to as "*clay shale*" when the predominant mineral constituent is clay. The shale in this presentation is not a source for oil or kerogen, but is shale or mudstone that has been compacted by overburden to the degree that any form of

porosity has become noneffective, and migrating oil cannot or has not penetrated the void space.

The term "connate", con-nate, often misused and ill-defined in petrophysics literature and glossaries, is from the Latin meaning: together at birth. In petrophysics it means together at time of deposition. Connate water is water entrapped within the pores or spaces between the grains or particles of rock minerals, muds, and clays *at the time of their deposition*. The water is derived from sea water, meteoric water, or ground surface water. Other investigators have shown that as clay shales and mudstones are compacted, a fresh water component of the original connate water is expelled and an ion-concentrated component remains in the voids. Therefore, water saturation in the voids remains 100% and S_w is 1.0 in ϕ_{ne} .

The relationship in Eq.(1a) below, presented in Ransom (1977), was developed to show that conductivity in water-filled voids in the host rock and the conductivity in clay shale can be represented by parallel conductivity relationships commonly used in basic physics. The relationship applies to heterogeneous rocks with a uniform distribution of minerals and porosity. In resistivity and conductivity relationships the dimensional units for a reservoir bed (m/m²) must be made to be unity so that bed dimensions will not be a factor.

For shaly sand it was shown that

$$1/R_{we} = (\phi_e/\phi_t)1/R_w + (\phi_{ne}/\phi_t)1/R_{wb} \quad (1a)$$

where $\phi_t = \phi_e + \phi_{ne}$.

For 100% water-bearing rock the conductivity relationship in Eq.(1a) yields $1/R_w$ (in clean rock) where ϕ_{ne} is 0.0, and yields $1/R_{we}$ (in shaly sands) when conductive clay is present and ϕ_{ne} is greater than 0.0.

It should be noted that in this equation there are no limitations on the resistivity values of formation water R_w and bound water in shale, R_{wb} . However, nature does set limits. The bound water resistivity R_{wb} in the mudstone and clay shale is related to the original connate water. Waters entrapped at the time of deposition do vary, but usually do not vary greatly from shale to shale. Sea water has an average salinity of about 35,000 ppm, other surface waters probably are significantly lower. However, as indicated above, the ion concentration in clay-bound water can increase as fresh water is expelled from the clays with increasing depth and compaction.

Unlike connate water, interstitial-water resistivity R_w in the reservoir can vary over a wide range. Interstitial water might have undergone many changes through the dissolution and/or

precipitation of minerals throughout geologic history. It can vary from supersaturated salt solutions to very fresh potable waters. Waters in the Salina dolomite (Silurian) in Michigan have specific gravities of 1.458 and a reported salinity of 642,798 ppm. In other formations, the waters vary to as little as a few hundred parts per million. Most often R_w will be greater than R_{wb} ; and R_{we} , a mixture of R_w and R_{wb} , will be lower than R_w . But, it is quite common for R_w to have a lower value than R_{wb} , in which case R_{we} will have some value greater than R_w .

In water-filled dirty sands, Eq.(1a) applies. In clean sands, Eq. (1a) becomes

$$1/R_{we} = (\emptyset_e/\emptyset_t)1/R_w = 1/R_w$$

and the corrected R_0 in either event is

$$R_{0 \text{ corrected}} = F_t R_{we} \quad \dots (2a)$$

where, from Figure 1

$$F_t = 1.0/(S_{wt}\emptyset_t)^{m_2} = 1.0/(\emptyset_t^{m_1}) \quad \dots (3a)$$

For the value of $R_{0 \text{ corrected}}$ the value of $S_{wt}\emptyset_t$ becomes \emptyset_t because S_{wt} has become equal to 1.0. Therefore, the exponent m_2 becomes m_1 as described above and in Figure 1. In Eq.(3a), the F_t that formerly pertained to $S_{wt}\emptyset_t$ now pertains only to \emptyset_t .

However, in the presence of oil or gas, water saturations are lower than 1.0 and the conductive water-filled volumes are reduced by the displacement of water volume by the hydrocarbon, and Eq.(1a) becomes

$$1/R_{we} = ((S_{we}\emptyset_e)/(S_{wt}\emptyset_t))1/R_w + (\emptyset_{ne}/(S_{wt}\emptyset_t))1/R_{wb} \quad \dots (1b)$$

and
$$R_{t \text{ calculated}} = F_t R_{we} \quad \dots (2b)$$

where R_{we} is determined from Eq.(1b), and

$$F_t = 1.0/(S_{wt}\emptyset_t)^{m_2} \quad \dots (3b)$$

Here S_{wt} is less than 1.0 and exponent m again becomes m_2 . And, it will be shown in **APPENDIX (B) (3)** that

$$(S_{wt}\emptyset_t)^{m_2} = (S_{wt})^n (\emptyset_t)^{m_1} \quad \dots (3c)$$

As it will be seen later Eq.(3b) is the key in the development of dual-water dual-porosity interpretations. Equation(3b) is a direct progression from the model in Figure 1 and will be further corroborated in the development from the common electrical resistance equation, Eq.(3d), that will be developed later.

Now that the importance of R_{we} in the above equations has been established, just what is R_{we} and can waters represented by R_w and R_{wb} actually be combined as seen in Eq.(1b)?

Water with resistivity R_w is interstitial water. Water with resistivity R_{we} is not interstitial water and does not exist in nature at 100% water saturation, i.e. as $R_{0 \text{ corrected}}$. Resistivity $R_{0 \text{ corrected}}$ is a hypothetical water mixture of dual waters, influenced by the presence of oil, that exists at 100% water saturation only in a dual-water concept. In electrical equations the dual waters can be mixed together, physically and in nature they cannot. A dual-water mixture with resistivity R_{we} cannot be recovered in a production test.

To continue, a very similar relationship to Eq.(2b), involving Eqs.(1b) and (3b), is seen in work published by Best et al (1980) and by Schlumberger (1989).

[An historical retrospection: Equations(1b) and (3b) result from early developments in resistivity well-log interpretation. Equation(1a) above was shown in Ransom (1977), and in the presence of hydrocarbon becomes Eq.(1b). The model in Figure 1 in this paper was first seen and described in Figure 2 on page 7 of Ransom (1974). From that model, F_t in Eq.(3b) above was first developed. Formation factor F_t of Eq.(3b) was shown as F_3 in the explanation of Figure 2, page 7, of Ransom (1974). The equivalence relationship, $(S_{wt}\phi_t)^{m_2} = (S_{wt})^n (\phi_t)^{m_1}$, is helpful for the use of Eq.(3b) in the determination of Archie's dual-water saturation. Although the proof of the equivalence is found in APPENDIX(B)(3) in this paper, the equivalence was first seen as Equation(8) on page 7 of Ransom (1974). Archie's saturation equation of 1942 was first derived in Ransom (1974) on pages 7 and 8. Also, dual porosity was suggested in Ransom (1974), where ϕ_e was described as hydrodynamically effective porosity and total porosity, ϕ_t , was described as electrically effective porosity. The difference between the two is hydrodynamically noneffective porosity, ϕ_{ne} . It was shown how to calculate all three porosities in Ransom (1977). Archie's relationships of 1942 were derived in Figure 2 of Ransom (1974), again in Ransom (1995), and again, in considerably greater detail, in Figure 1 in this paper.]

All equations referred to and all equations appearing in this paper support the fundamental certainty of the influence of the geometry of the bulk volume of water on the electrical resistance of rock. The bulk volume geometry concept was first introduced relative to resistivity analysis in Ransom (1974). This concept is best exemplified by the bulk volume of water term, $S_{wt}\phi_t$, in Eqs.(1b) and (3b), and by water geometry represented by exponent m_2 , and therefore by both m_1 and n , observed in **Figure 1** and seen in Eq.(3b).]

In Eq.(3b) both the bulk volume of water, $S_{wt}\phi_t$, and its electrical efficiency can be seen to change in the term $(S_{wt}\phi_t)^{m_2}$ as the saturation distribution of hydrocarbon changes the geometry of water. The basic Archie method is changed from a single-water single-porosity method to the dual-water dual-porosity methodology where the two waters have resistivities of R_w and pseudo R_{wb} . The former R_w of the single-water single-porosity method becomes R_{we} , as seen in Eqs.(1a) and(1b). The R_{wb} has been referred to as a pseudo R_{wb} because some of the conductivity attributed to the clay shale might not be in the form of water, but the usual resistivity-measuring logging tools do not know the difference.

In dual-porosity dual-water methodology the numerator in Eq.(3b) always must be 1.0 because the compensation for all electrically conductive influences, along with formation water R_w , should be relegated to terms in the R_{we} equation.

Equation (3b) applies to all cases where $S_{wt} \leq 1.0$, and the term $(S_{wt}\phi_t)$ in the denominator of the F_t relationship must be compatible with the same term in the denominator in the calculated R_{we} relationship in Eq.(1b) in whatever application it is used. The fractional volume of conductive water represented in the formation factor must always be the same fractional volume of water that exhibits the equivalent resistivity R_{we} . Also, note that both S_{wt} and ϕ_t have the same exponent m . In this case, the graphical model shows this m to be m_2 because m_2 is the combination exponent equivalent to the individual exponents m_1 and n . It is illustrated in the trigonometry of Figure 1 that the same result would be realized if S_{wt} and ϕ_t were to use their individual exponents n and m_1 , respectively, because calculated water saturation is related to the resistivity level of R_t and this remains unchanged. The algebraic proof that $(S_{wt}\phi_t)^{m_2} = (S_{wt})^n(\phi_t)^{m_1}$ is shown in **APPENDIX (B) (3)**.

In Figure 2, adapted from Ransom (1974), it is illustrated that the resistivity value of water-filled rock at any specific depth can be the result of a shift by the influence of a variable a , and the cause of that influence must be explained.

In these equations the accountability or compensation for the influence of clay shale in shaly sands resides in the proportionality terms involving the conductivity counterparts of R_w and R_{wb} observed in Eq.(1b). In this relationship the proportionality terms are functions of both porosity and saturation as well as clayiness. It further can be seen that for every value of clayiness or "shaliness" and resulting volume of bound water within ϕ_{ne} , the prevailing values of effective porosity and water saturation can vary the conductive water volume in effective pore space and, therefore, can vary the relative proportions of R_w and R_{wb} for the mixture R_{we} . And, this can and does vary depth by depth.

It is further illustrated in **Figure 2** that as R_w is corrected to R_{we} , R_0 is corrected to $R_{0 \text{ corrected}}$, both by the same factor a . Just as ϕ_e and ϕ_t are two intrinsic properties of the rock and exist simultaneously, R_w and R_{we} exist simultaneously and relate to the same exponent m . The m exponent describes an intrinsic property of the rock and produces the parallelism seen in the Figure 2. As a result, the slope represented by m is independent of the conductivity of the waters in the pores. In this figure, R_w is related to R_{we} by the factor a , or coefficient a , that varies depth by depth. As a consequence, when these relationships for R_{we} are implemented, or their derivatives or equivalent relationships are used in any resistivity-based interpretation, it is important to recognize that the accountability or compensation by the a coefficient must be removed from the modified Formation Resistivity Factor. And, the numerator of the Formation Resistivity Factor must always be equal to 1.0 to prevent duplication in the accountability for the secondary conductivity provided by clay shale or other conductive constituents. This perspective will be discussed in detail below.

WHAT IS THE FORMATION RESISTIVITY FACTOR

The Formation Resistivity Factor, F , is an intrinsic property of a porous insulating medium, related to the degree of efficiency or inefficiency for the electrolyte-filled paths to conduct electrical current through the medium. The formation factor pertains to and is intrinsic to the insulating medium only. It is independent of the electrical conductivity of the electrolyte in its pores. Any recognizable, valid extraneous electrical conductivity that sometimes is seen to influence the value of the formation factor must be relegated to appropriate conductivity equations, such as Eq.(1a) and Eq.(1b), where conductivities can be accommodated by discrete terms.

An equation that demonstrates the purpose of the formation factor is the very basic resistance equation that converts resistivity to resistance:

$$r \text{ ohms} = ((L/A)m/m^2) (R \text{ ohms } m^2/m) \quad \dots(3d)$$

where r is resistance, L is length, A is the cross-sectional area of a straight electrically-conductive path of length L , and R is the familiar resistivity. The term L/A in the equation is similar to a formation factor and describes a fractional volume having length L and cross-sectional area A that is 100 percent occupied by a single, homogeneous, electrically-conductive medium, either an electrolyte or solid. This equation represents 100% efficiency in the conversion from resistivity to resistance, and the converse.

Figure 3 is similar to a figure used earlier by Ransom (1984,

1995). This figure represents a unit volume of insulating solid matter where each side of the cube has a length L equal to 1.0 meter. Remove 0.20 of the insulating cube from the center and fill the void with water of resistivity R_w . Electrical current now can pass from top to bottom of the cube through the water without deviation or interference. This form of electrical path through the insulating matter has the highest efficiency.

A measurable resistance implies a measurable resistivity, and resistivity implies conductivity. And, within this insulating cube, the only electrical conductivity occurs in the fraction that is water. Therefore, the measured resistance across the example cube is due to the resistance of the water. Eq.(3d) now can be written

$$r_{cube} = (r_{water} \text{ ohms}) = ((L/0.2L^2)m/m^2) (R_w \text{ ohms m}^2/m)$$

If this cube were a unit volume of rock where all void volume was represented by 20% porosity, then, the relationship would be

$$r_{rock} = R_{water} = (L/0.2L^2) (R_w) = (1.0/0.2L) (R_w)$$

The dimensional analysis of this relationship is

$$(\text{ohms}) = (m/m^2) (\text{ohms m}^2/m) = (\text{ohms})$$

and the *apparent* formation factor is $(1.0/0.2L)$ with units of reciprocal meters, (m^{-1}) . After dividing both sides of the equation by the volume represented by L/A , as observed in Eq.(3d), resistance becomes resistivity and the relationship becomes

$$(A/L) r_{rock} = R_{rock} = R_t = (A/L) (1.0/0.2L) (R_w)$$

Here, the dimensional analysis is

$$\begin{aligned} (m^2/m) (\text{ohms}) &= (m^2/m) (1/m) (\text{ohms m}^2/m) \\ (\text{ohms m}^2/m) &= (m^2/m^2) (\text{ohms m}^2/m) = (\text{ohms m}^2/m) \end{aligned}$$

and the formation factor now is dimensionless and has become

$$F = 1.0/(0.2) = 1.0/\phi_t = 1.0/\phi_t^m$$

in its simplest form, where exponent $m = 1.0$.

In addition to showing the relationship between the formation factor and the void space in the rock, this exercise demonstrates that the absolute minimum value for exponent m or m_1 is 1.0 at the highest possible electrical-path efficiency of 100%.

Theoretically, this degree of efficiency can be duplicated by the presence of an open fracture or other similar water-filled void aligned favorably with the electrical-survey current flow.

Although the value of m might never reach 1.0 in practice, the presence of fractures and similar voids can and do reduce the value of m .

There is a fine distinction between the formation factor that is an intrinsic property of rock related to the shape of its voids, and the formation factor that is related to the shape of the water that occupies the voids. The intrinsic formation factor is directly related to the solid insulating framework of rock and how it shapes the electrically-conductive water volume when water saturation is 100%, and nothing else. This by definition is intrinsic. But, the size and shape of the conductive water volume also is influenced by the presence of insulating fluids, such as oil and gas, that can displace water and occupy part of that pore volume. Therefore, the distinction is made that the actual formation factor, that will be called F_t , used in calculations and derivations, will involve the term S_{wt} . At any saturation other than 100%, F_t no longer is intrinsic.

To carry this demonstration one step further, in the same rock where $F = 1.0/\emptyset_t$ at 100% efficiency, if part of the rock is electrically inert, heterogeneous, porous, insulating rock framework, with a uniform distribution of constituents and porosity, and the remainder is formation water partly displaced by hydrocarbon, then the former volume of water \emptyset_t now becomes $S_{wt}\emptyset_t$ and F now becomes $1.0/(S_{wt}\emptyset_t)$ for all values of water saturation and porosity. The fraction $S_{wt}\emptyset_t$ now has become the fractional cross-section of area in **Figure 3** for all electrically-conductive water paths where the electrical efficiency is 100%.

Not all water-filled electrically-effective pore paths are 100% efficient because the network of interstitial water within the rock framework can take on many different shapes and configurations imposed by the many and varied properties of the pore walls and rock framework. Where $S_{wt}\emptyset_t$ is the fractional cross-section area for all electrically-conductive water paths at their highest efficiency, as in a bundle of straight tubes, $(S_{wt}\emptyset_t)^{m_2}$ is an equivalent cross-sectional area resulting from all factors that impede the flow of electrical current or increase the resistance-to-flow through the rock. This interference to electrical-current flow is reflected in the steepness of the slopes of the tangential exponents m_2 , m_1 , and n in **Figure 1**. As a result, $1.0/(S_{wt}\emptyset_t)$ becomes $1.0/(S_{wt}\emptyset_t)^{m_2}$ to accommodate the varied geometries of pores and paths for all conditions of pore path inefficiency and interference, and the Formation Resistivity Factor once again becomes

$$F_t = 1.0 / (S_{wt}\emptyset_t)^{m_2} \quad \text{from (3b)}$$

the same as it was derived from the trigonometrics in triangle AEG of Figure 1.

The Formation Resistivity Factor is aptly named. Resistance r of the interstitial-water network, and therefore resistivity R_t of the rock, is a function of the size and geometric dimensions of configuration and shape imparted to the water volume within the network of interconnected pores. In one unit of total volume, size or cross-sectional area through which electrical current must flow is related to both porosity and water saturation as a fraction of that one unit area. Tortuous length, configuration, saturation distributions, and shape of the electrical pathway filled with water volume $S_{wt}\phi_t$ determine the efficiency or inefficiency of in-place water to conduct electrical current and, therefore, provide a value for exponent m_2 .

In this conversion of formation water resistivity to pathway resistance, the greatest efficiency at any given value of $S_{wt}\phi_t$ occurs when exponent m or m_1 equals 1.0 and m_2 approaches 1.0. And, the greatest effectiveness in one unit of total volume occurs when both S_{wt} and ϕ_t are 100% or 1.0. When that happens, $F_t = 1.0$ and $r = R = R_w$. At any given saturation, overall efficiency of water paths decreases as ϕ_t decreases and/or as m_1 , m_2 or n increases.

All factors that influence the parameters referred to in this discussion determine the efficiency for electrical-survey current to flow through the rock. These factors, through Archie's parameters, ultimately determine the value given to the Formation Resistivity Factor. Next, we look at those factors.

THE m EXPONENTS

The porosity exponent m is an intrinsic property of the rock related to the geometry of the electrically-conductive water network imposed by the pore walls or surfaces of solid insulating materials. This has been verified by investigators of porous media in extremely meticulous and controlled laboratory investigations, particularly by Atkins and Smith (1961). All minerals have characteristic crystalline or particle shapes, whether the minerals are electrically conductive or not, and contribute to the geometry of the electrically conductive water residing in the pores through shape, physical dimensions of the pores and pore throats, tortuosity, configuration, continuity, pore isolation, orientation, irregularity, surface roughness, angularity, sphericity, and anisotropy. In addition, allogenic minerals, and authigenic mineral growths such as quartz or calcite or dolomite or clays, all, contribute to the water geometry within the pores. And, the electrical pathways through the insulating rock are configured further by secondary porosity in all its shapes and forms, such as: dissolution porosity, replacement porosity, fissures, fractures, micro-cracks, and vugs in their various orientations within the rock. Exponent m is related to all these features acting separately or in concert through the various

inefficiencies (or efficiencies as the case might be) imparted to the electrically-conductive paths by the diversified void geometries, and this results in variations in electrolyte-filled path resistance and consequent rock resistivity. The greater is the electrical inefficiency of the shape of the resulting water volume, the greater will be the value of exponent m , and the greater will be R_0 and/or R_t . This, too, can be seen in **Figure 1**.

Heterogeneous solid matter, if it is electrically inert, constitutes part of the framework of the rock only and has no other effect on resistivity other than by occupying a position in the rock framework and possibly displacing interstitial water and influencing the pore shape and pore geometry.

Heterogeneous electrically-conductive minerals such as pyrite and siderite present a different scenario. Their presence not only can affect the shape of the conductive water paths in the same manner as electrically inert minerals, but can provide electrical conductance in solid matter, not related to pore geometry, that can influence rock resistivity. The electrical conductivity of such minerals must be accommodated through the a coefficient in proportionality relationships in Eq.(1a) and (1b), or an equivalent.

In practice, the m exponent usually has a default value of about 2.0. In the laboratory, the minimum value for m (or m_1) in homogeneous granular media has been determined to be about 1.3 for spherical grains regardless of uniformity of grain size or packing (Atkins and Smith, 1961; Fricke, 1931; Pirson, 1947; Wyllie and Gregory, 1952). It was shown in the development of the formation factor, above, and in Ransom (1984, 1995), that m will decrease in the presence of an open fracture, dissolution porosity, or fissure where the continuous void space is aligned favorably with the survey-current flow; and it was further demonstrated that m has an absolute minimum value of 1.0 at 100% efficiency.

In the concept in this paper, the porosity exponent m_1 can be estimated from rock containing 100% water by

$$m = (\log R_0 - \log R_w) / (\log 1 - \log \emptyset_e) \quad (5a)$$

and
$$m = (\log R_{0 \text{ corrected}} - \log R_{we}) / (\log 1 - \log \emptyset_t) \quad (5b)$$

The R_{we} in Eq. (5b) is from Eq. (1b) and reflects the change in relative proportions of R_w and R_{wb} as they are altered by the occupation of oil or gas. Although the slope m is intrinsic and the m from Eqs. (5a) and (5b) should be the same, it is slope m from Eq. (5b) that is seen in Figure 2.

Additionally, a value for m_1 can be determined that applies to a larger range of data. Turn to **Figure 5**. This figure is a plot of

R_{wa} vs *Clayiness*. The assumption is made that ϕ_t , R_t , and *Clayiness* are reliable. The value R_{wa} is determined by dividing R_t by the formation factor in Eq.(3a). In interactive computer methods, the plot is entered by assuming a reasonable value for the m exponent. Select an interval that is believed to contain some wet zones, if possible. Most often an R_{wz} trend can be recognized in this plot, as seen in **Figure 5**. Both an R_w and R_{wb} often emerge. If an independent R_w value is known from a reliable source, convert that R_w to the in situ temperature of the logging tool environment at the depth of interest. If the R_w from the plot differs from the known value, iterate by program subroutine between the known R_w and the derived R_w by incrementally changing m in Eq.(3a) until the two values agree. The value of m determined in this manner is compatible with the log data for evaluation of R_w and R_{wb} .

In the event that the m of Eq.(5a) does not agree with m in Eq.(5b), other methods for singular values of m are based on **Figure 2**. One method is based on the solution of m in similar triangles, and another is based on the dual salinities of R_w and R_{we} in a method similar to the laboratory method proposed by Worthington (2004). Both methods require considerable iteration in the evaluation of unknown variables in their respective equations.

It should be noted that on a log-log plot for either measured or corrected values of R_0 (or C_0) versus ϕ_t , as in Figure 2, values of m sometimes can be derived for each specific set of log data or rock sample. Individual values of R_{we} and $R_{0\text{ corrected}}$ will continually change with changing clay shale content and water saturation. As a result, a reliable trend specific to comparable rock samples might not be observed for m unless the ratio of ϕ_{ne}/ϕ_t , or other appropriate discriminator, is held nearly constant to ensure a nearly constant relationship between the proportions of R_w and R_{wb} as porosity changes. And, laboratory measurements for m must show repeatability in both m and the resistivity of the influent and effluent after significant time lapses to prove that the measurement process, or sample degradation, does not influence the value of the measurement being made.

In the single-exponent method for determination of water saturation, it was seen in triangle AEG that:

$$m_2 = (\log R_t - \log R_{we}) / (\log 1 - \log(S_{wt}\phi_t))$$

The value of the bulk volume water exponent m_2 is not an intrinsic property of rock, as exponent m . It is a hybrid value due to the presence of oil or gas.

HOW IS EXPONENT n RELATED TO EXPONENT m

The saturation exponent n is the most misunderstood parameter in

formation resistivity interpretation. But, its purpose in electrical resistivity interpretation is quite straightforward, as is the purpose of exponent m . The saturation exponent n is related to both pore geometry and the interference to electrical current flow within the complex water-filled paths remaining in the pores *after displacement by hydrocarbon has taken place*. In Figure 1 and Eq.(3b) it is shown that n is a geometrical element similar to m . In **Figure 1**, it can be seen that *slope n is what slope m becomes after hydrocarbon has migrated into the pores and has displaced a fraction of the water.*

Historically, and in the absence of better information, the usual default value for n in water-wet- and oil-wet rocks has been the same as for m ($n = m$), i.e. exponent m_1 . Also, see discussion in **APPENDIX(B) (4)**. It has been cited in petrophysics literature that n often is $< m$. However, the presence of insulating oil at any saturation displaces some water volume and produces some electrical interference and, therefore, the value of n must be greater than m and must decrease the electrical effectiveness of the remaining bulk volume of water. The presence of oil cannot increase the electrical conductivity of water to a value more than 100%, and can neither increase the volume of electrically conductive water paths nor increase the effectiveness of the electrically effective water paths more than actually exists. Much to the contrary. Because of increased electrical interference by the presence of hydrocarbon, the result will be to increase the minimum value of n to some value higher than the value of m , all other things remaining the same. Because n is what m_1 has become after hydrocarbon has displaced a fraction of water, exponent n cannot have a lower value than the actual value of m_1 . A detailed explanation appears in **APPENDIX(D)**, based on **Figure 6**, why the actual value of n cannot be lower than the actual value of m_1 in the same sample, in situ or in the laboratory. Exponent n can and will increase over m_1 in the presence of oil or gas as the presence of hydrocarbon decreases the volume of electrically conductive water and changes the dimensions of electrical paths, or otherwise impedes the flow of the electrical-survey current. Additionally, there can be multiple values for both m_2 and n as oil saturation changes and/or wettability to oil changes. For oil-wet and partially oil-wet rocks this effect can be quite significant. When oil is present, in partially oil-wet and oil-wet rocks, for any given water saturation the saturation exponent n can vary from as low as m to as high as 8.0 or 9.0 or more depending on the degree of, and effectiveness of, wettability to oil, physical distributions of oil and water, oil properties, and rock-framework surface properties and characteristics. In **Figure 1**, it can be seen that at any constant value of R_t , if the redistribution of a constant fraction of oil causes the electrical interference to change, then the slope representing exponent n (and m_2) will rotate to a different intercept H with R_t , and this will result in a corresponding change in the value of S_{wt} .

The exaggerated influence due to the presence of oil will increase both the usual exponent n for S_{wt} and the combination exponent m_2 for bulk volume water $S_{wt}\phi_t$. For any given porosity and any given oil saturation, the slopes for exponents n and m_2 will increase with those properties of the rock and pore walls that when covered with adhesive oil films increase the interference to the flow of electrical current through the conductive paths. These factors increase in severity with the increase in wettability to oil, finer grained sandstone (increased surface area), increased efficiency of packing, increased number of grain-to-grain contacts, finer pores and pore throats, properties of oil (increase in viscosity of oil), interfacial tension between oilfield brine and crude oil, isolation of pores, and the physical saturation distributions of both the wetting- and nonwetting-phases whether oil or water. All these influences act in concert at their respective levels of severity to cause or alter interference to electrical current flow.

In oil-wet and partially oil-wet rock, the effects of these factors become magnified and the electrical interference within the pore paths is increased. As a result, the saturation exponents n and m_2 increase. In oil-wet rock, it might be thought that the higher the value of these exponents, the higher will be the oil saturation. To emphasize, that is not the case. Figure 1 shows that the saturation exponent n represented by slope CG of triangle CDG is the resistivity gradient employed between R_0 corrected and R_t relative to changes in the saturation of oil (or water). Similarly, exponent m_2 is the gradient between R_{we} and R_t relative to changes in bulk volume water, $S_{wt}\phi_t$.

The effectiveness of the cumulative interference is related to the existing water saturation. Observe the curves in **Figure 10**. It can be seen that interference is more effective at higher water saturations.

The maximum value for n in any specific water-wet rock is that value where the presence of oil or gas has the greatest effectiveness in producing electrical interference. This occurs at high water saturations. Interference and how effective that interference can be are not the same thing. The effectiveness of the cumulative interference to increase resistivity is greatest at higher water saturations, and is least effective to increase resistivity at low water saturations. Read that carefully. Again, study the effectiveness curves in **Figure 10**. The greatest electrical interference occurs in rock with the greatest wettability to oil and depends on the distribution of the oil and its viscosity. The minimum value for n occurs where the interference caused by the presence of hydrocarbon exhibits the least effectiveness in producing electrical interference. The greater the value of n or m_2 for any given value of resistivity, the greater will be the calculated water saturation. But, these

statements should not imply that there is a strong relationship between the value of n or m_2 and the value of either water or oil saturation. There is none. That is because of the many different factors that can influence the presence and distribution of oil and its properties, and thereby influence R_t . There is no mathematical relationship between the value of exponents n or m_2 and the value of oil saturation, except when within the same bed with uniform electrically effective constituents. To the contrary, it will be seen later in this paper, and can be seen in Figure 1, that if all other things remain constant, as exponent n increases along angle γ (or δ) or m_2 increases along angle β , water saturation increases with the possibility of increased water cut; and/or, water production takes place without significant oil. In the model, it can be seen that when R_t is constant, if n or m_2 increases, the slope representing n or m_2 becomes more vertical (n might increase to 8 or 9) and the downward projection of slope n is toward increased water saturation, S_{wt} , and the downward projection of m_2 is to an increased bulk volume of water, $S_{wt}\phi_t$.

That having been said, it must be understood that the two exponents n and m_2 not only represent gradients, but represent efficiency or effectiveness. And, effectiveness of oil to interfere with the flow of electrical survey current is greatest at high water saturations where oil saturation is the lowest. This is corroborated by the slope CG representing exponent n in **Figure 1**. Is this contrary to anything said above? It might appear so, but in actuality, it is not.

These features relative to the presence of oil, and sometimes gas, must be recognized. Is there any exception? Theoretically, it might be possible to hypothecate a condition whereby the value of n could have a value lower than m , but it is not likely. In literature, the value of exponent n with a value lower than m is commonplace and overwhelmingly is accepted as conventional wisdom; but, it violates physics and it has never been explained in the same literature how a valid $n < m$ can occur. In the laboratory $n < m$ is a common occurrence due to sample damage or degradation. In situ, $n < m$ would be a violation of physics.

See discussion under **APPENDIX (D)**. For a comprehensive treatment of n , please study the explanations under **OBSERVATIONS AND CONCLUSIONS FROM FIGURE 10 ABOUT EXPONENT n** . The discussions following the Observations and Conclusions absolutely corroborate triangle CDG of the model in **Figure 1**.

Gas usually does not have the same exaggerated effect on n as oil unless the reservoir has been filled with oil at some former time in geologic history and an adhesive film of remnant oil precedes the occupation by gas. The resistivity of a gas-bearing zone can increase, however, due to the decrease in irreducible water saturation. This, too, can be demonstrated in **Figure 1**. The

primary exaggeration in n is with partially oil-wet and oil-wet rocks that are filled with oil or have been filled with oil at a former time whether as a reservoir or as a migration path.

THE a COEFFICIENT

Historically, the a coefficient always has appeared in the numerator of the modified Formation Resistivity Factor, and has been perpetuated in industry with no defined purpose. In this model there is no support for the appearance of an a with a constant value in the formation factor. The formation factor is intrinsic to the rock at 100% water saturation. The a coefficient is not an intrinsic property, but is dependent on variables not related to the electrically inert rock or to the physical geometries of its pores. The a is related neither to volume nor shape imparted to the network of electrically-conductive water paths. The a coefficient contributes nothing to alter efficiency of the conversion of water resistivity to resistance, or vice versa. In the model, the shift in resistivity corresponding to a always is found in the resistivity axis, and the shift varies from data to data with depth. Coefficient a emerges as an inherent and inseparable factor of the complex resistivity relationship, as in Eqs. (1a) and (1b), and is related to the proportions of all secondary electrically-conductive constituents and influences that vary the relative proportions of R_w and R_{wb} .

It can be seen in the graphics of **Figure 2** that R_{we} is the product of a and R_w , therefore $aR_w = R_{we}$. R_{we} varies from depth to depth, and so does coefficient a . It can be seen in this relationship that a varying a coefficient technically can be used in single-water single-porosity methods where R_{we} is not calculated. But, in dual-water dual-porosity methods where R_{we} is calculated, the a coefficient cannot be used as an independent parameter. If the a is used in single-water single-porosity methods, it must be calculated properly. And that is difficult to do when the correct value of a varies with rock constituents and depth, and only one water and one porosity is available to work with. That is one of the reasons why dual-water dual-porosity methods were conceived.

When a is associated with R_w , as in the equation developments of (1a) and (1b), and as a multiplier or reduction factor for R_w , the a coefficient more readily can be recognized as being an indispensable factor that accounts for those heterogeneities that produce additional electrical conductivity. Coefficient a is the composite factor that relates R_w to the resistivity value of the combination of R_w and R_{wb} , *and all other natural electrically-conductive influences*, in such proportions to result in R_{we} .

In the modified formation factor used in single-water single-porosity methods, the a coefficient appeared in the numerator. In

the modified formation factor the a became a multiplier of the R_w in Archie's saturation equation. The aR_w became a single-water single-porosity equivalent to R_{we} .

Secondary conductive influences, inherent to the rock, that produce coefficient a can be: clay shale, surface conductance, or solid semi-conductors such as pyrite and siderite as separate influences or in concert. One influence that is neither electrically conductive nor intrinsic is hydrocarbon saturation. These in turn, and perhaps others, can produce a change in rock resistivity.

The occurrence of one additional electrically-conductive influence would add one more term to Eqs. (1a) and (1b), or their equivalent, and if there were no change in $S_{wt}\phi_t$, the value of the compound coefficient a would be reduced. These influences, all, can change R_w to an apparent or pseudo value R_{we} with a corresponding change in R_0 to $R_{0 \text{ corrected}}$ at their respective porosities.

Technically, in dual-water dual-porosity methods, the coefficient a should be transposed from the modified Formation Resistivity Factor term to the water resistivity relationship. For example:

$$R_{0 \text{ corrected}} = F_t R_{we} = (a / (\phi_t^m)) R_w = (1.0 / (\phi_t^m)) a R_w = (1.0 / (\phi_t^m)) R_{we}$$

The question might be asked, "What is the difference?" There are a number of reasons, four of which are:

1. The a coefficient is a required proportionality factor, related to all factors influencing the electrical conductivity of the rock, that convert resistivity R_w to R_{we} , and is calculated along with R_{we} at each depth increment.

2. To prevent duplication by the user in the correction for conductive influences. Duplication occurs when both a and R_{we} appear in the same water-saturation equation, or when a appears as an independent variable in dual-water dual-porosity methods. Any saturation relationship involving resistivity can employ either coefficient a or R_{we} , but not both.

3. To prevent the use of a constant artificial and unwarranted correction factor.

4. Although the formation factor is meaningless at 100% porosity, the value of F_t always must be 1.0 when both porosity and water saturation in the F_t equation are 1.0.

As explained above, and in Figure 2,

$$a = R_{we} / R_w \tag{1c}$$

After substituting Eq.(1c) into Eq.(1b), a relationship is derived showing the proportionality terms in coefficient a for the usual shaly sand:

$$1/a = (S_{we}\phi_e)/(S_{wt}\phi_t) + (\phi_{ne}/(S_{wt}\phi_t)) (R_w/R_{wb})$$

It can be seen here that coefficient a is a function of water saturation as well as porosities. See **APPENDIX (A)** for further explanation. This is the evaluation for the a coefficient appearing in **Figure 2**.

This relationship is shown for comparison purposes or informative purposes only. It is not to be calculated and used independently in dual-water dual-porosity methods because it already has been incorporated in the calculation of R_{we} as can be seen in Eq.(1b).

THE SATURATION EVALUATION

The model in Figure 1 is a diagram showing that R_t is a function of both the volume of water $S_{wt}\phi_t$ and the inefficiency with which electrical current passes through that water. The inefficiency of the electrical current flow is related to the distribution of the water and the interference to that flow within the water network as it is reflected in the exponents m and n of the expression $(S_{wt})^n(\phi_t)^m$. In the model it is shown that $\log S_{wt}$ is the length of the projection along the X-axis between $\log \phi_t$ and the intercept of slope n with $\log R_t$. This length also is represented by length CG of triangle CDG in **Figure 1**.

Revisiting Eq.(1b), (2b), and (3b), the reader already might have deduced that water saturations can be estimated from these equations. Keeping faithful to the self-evident truth that the volume of water referred to in the denominator of the formation factor must be the same volume of water that provides electrical conductivity in the R_{we} equation, then Eq.(1b) can be used only with Eq.(3b). Therefore, when S_{we} (or S_{wt}) is less than 100%, the product resulting from Eq.(1b) and (3b) is

$$R_t \text{ calculated} = F_t R_{we} \quad \text{same as (2b)}$$

where $F_t = 1.0/(S_{wt}\phi_t)^{m2}$ from (3b)

After combining Eq.(2b) and (3b) when either the measured or *actual* R_t is substituted for the *calculated* R_t , then

$$R_t \text{ measured} = (1.0/((S_{wt}\phi_t)^{m2})) a R_w = (1.0/((S_{wt}\phi_t)^{m2})) R_{we} \quad \dots (4a)$$

However, it was illustrated in **Figure 1** in triangle ACG, and in **APPENDIX (B) (3)**, that $(S_{wt}\phi_t)^{m2}$ is equivalent to $(S_{wt})^n(\phi_t)^{m1}$, as seen in the equivalence equation (3c) therefore Eq. (4a) resolves to

$$S_{wt}^n = (1.0/(\phi_t^{m1})) (R_{we}/R_{t \text{ measured}}) \quad \dots (4b)$$

Archie's dual-water dual-porosity equation.

It was passed over quickly that Equation (4a) also yields Archie's equivalent single exponent version seen here

$$S_{wt}^{m2} = (1.0/(\phi_t^{m2})) (R_{we}/R_{t \text{ measured}}) \quad \dots (4d)$$

Only one R_{we} equation can be used in this calculation. It will be from Eq.(1a) or from Eq.(1b). At $S_{wt} = 1.0$, the calculations for R_0 are the same in either equation if the water mixture is the same. At $S_{wt} < 1.0$ only Eq.(1b) should be used for calculating R_t because it is the only equation that allows the calculated R_{we} to reflect the changing proportions of R_w and R_{wb} resulting from the displacement of interstitial water volume by oil or gas. In Eq.(1a) the proportions of the two waters are fixed by the mineral constituents of the rock. But, the relative proportions of the two waters and their electrical efficiencies also change with the change in saturation and distribution of oil and gas, and these change depth by depth. The $R_0 \text{ corrected}$ must be determined from the same water mixture proportions and water geometry that exist at each R_t measurement. These proportions are reflected only in the conductivities shown in Eq.(1b). The efficiencies are reflected in the exponent residing in F_t , and the F_t used must be compatible with each variation in water saturation, and that is Eq.(3b).

For Eq.(4b), Eq.(1b) can be simplified after the substitution for $S_{we}\phi_e$ has been made from the volumetric material balance equation for water,

$$S_{wt}\phi_t = S_{we}\phi_e + (1.0)\phi_{ne} .$$

After the substitution, Eq.(1b) becomes

$$1/R_{we} = 1/R_w + (\phi_{ne}/(S_{wt}\phi_t)) (1/R_{wb} - 1/R_w) \quad \text{simplified Eq.(1b)}$$

This version of R_{we} is used in Eq.(4b).

The $R_{t \text{ measured}}$ in Eq.(4b) must be corrected for environmental conditions and tool-measurement characteristics before water saturation is calculated.

The term S_{wt} in the formation-factor relationship of Eq.(3b) is the key element in the dual-water dual-porosity relationship. The term S_{wt} is an inherent part of the formation factor derived from the model in Figure 1.

It can be seen in Eqs.(1b) and (4a) that the a coefficient is variable with depth and mineralization and is included as part of R_{we} . When R_{we} has been calculated, and is used, the appearance of a constant a coefficient in the formation factor, usually as a

fraction less than 1.0, would be gratuitous and would artificially increase the calculated hydrocarbon saturation in productive and nonproductive zones alike; and, in this model, would be both logically and mathematically incorrect.

Figure 1, together with Figure 2, is a concept model that has significant informative and educational value. The graphics of the model are meant primarily to illustrate, to develop, or to explain what is calculated blindly by algebraics in computer-program subroutines.

In an interactive computer program, irreducible water saturation or other core-derived information can be input for the purposes of examining the plausibility, validity, and integrity of certain parameters. On a well log above the transition zone in oil-bearing reservoir rock, for example, the intersection of R_t with a laboratory value of irreducible water saturation fixes the upper limiting value of saturation exponent n for that specific set of data. However, when irreducible water saturation is known, this upper limit of exponent n should be calculated from the algebraics of Eq.(4b), or Eq.(4c) as will be shown below. The same can be said for the lower limit of n in the same rock which could be estimated by inserting water saturation when oil saturation is irreducible. But, in either exercise, no value of n can be lower than m_1 . The actual value of R_t is required for each of these procedures, whether derived from the well-log or rock sample.

The water saturation equation, Eq.(4b), has been developed from the trigonometric model in Figure 1 and again corroborated by the algebraic development of Eq.(4b), all, for certain heterogeneous, but uniform, environments. And, each development herein shows that it authenticates Archie's basic relationships presented in 1942, and further refines these relationships in the developments and discussions.

It has been said that Archie's relationships are empirical developments. Whether or not this is true, it has been shown here that Archie's classic relationships and parameters have a mathematical basis, and have forthright and substantive relevance to rock properties that is quite different from many accepted theories and usages in industry literature.

Saturation exponent n is the most difficult of all the parameters to evaluate. If a valid value of oil saturation is known, or can be derived, the value of exponent n can be estimated by substitution in Eq.(4b) or (4c). When the actual values of m and n are known, or can be derived, either or both can be important mappable parameters, and a mathematical relationship between m and n not only can be an important mappable parameter, but can be a possible indicator to the degree of wettability to oil or distribution of oil under in situ conditions. This information not only can be

important in resistivity log analysis but can be important in the design of recovery operations.

The calculation of Eq.(1b) is required in the solution of the water saturation relationship in Eq.(4b). Because water saturation S_{wt} also appears in the proportionality terms within the R_{we} equation, Eq.(1b), an algebraic solution for S_{wt} in Eq.(4b) is not viable and is not considered. Probably the simplest and best method for all anticipated integer and non-integer values of n is an iterative solution performed by a computer-program subroutine. Graphically the iteration process can be demonstrated by a system of coordinates where both sides of Eq.(4b) are plotted versus input values of S_{wt} . As S_{wt} is varied, the individual curves for the left and right sides of Eq.(4b) will converge and cross at the S_{wt} value that will satisfy the equation.

Figure 4 shows a crossplot of example data to demonstrate the equivalence of the graphical solution to the iterations performed by a computer-program subroutine. The following input values are for illustration purposes only.

$$\begin{array}{ll}
 m = 2.17 & R_w = 0.30 \\
 n = 2.92 & R_{wb} = 0.08 \\
 \phi_t = 0.22 & R_t = 20.00 \\
 \phi_{ne} = 0.09 &
 \end{array}$$

In **Figure 4** it can be observed that when values from each side of Eq.(4b) are plotted versus S_{wt} the two curves have a common value at a water saturation of about 0.485. The iteration by subroutine will produce the same S_{wt} of about 0.485 or 48.5% for the same basic input data.

It is worthy of note that in the volumetric material balance for water, when $S_{we}\phi_e$ goes to zero the absolute minimum value for $S_{wt}\phi_t$ in this example is 0.09, the value of ϕ_{ne} . The mathematical minimum water saturation S_{wt} that can exist in this hypothetical reservoir is $\phi_{ne}/\phi_t = 0.4091$ or 40.91% where R_{wb} becomes 0.08. The minimum saturation of 40.91% is related only to the pseudo bound water in clay shale and tells us nothing about irreducible water saturation in the effective porosity. If this were an actual case in a water-wet sand, at 48.5% water saturation, water-free oil might be produced because the only water in the effective pore space might be irreducible. Grain size and surface area would be a consideration. Any water saturation below 40.91% cannot exist and is imaginary.

For the conversion of S_{wt} to S_{we} , either the material balance for water (shown above) or the material balance for hydrocarbon (Ransom, 1995), can be used. In terms of hydrocarbon fractions, the material balance for the amount of hydrocarbon in one unit volume of rock is

$$(1.0 - S_{wt})\varnothing_t = (1.0 - S_{we})\varnothing_e$$

For the calculation of S_{we} the balance can be re-arranged to read:

$$S_{we} = 1.0 - (\varnothing_t/\varnothing_e)(1.0 - S_{wt}) \quad \dots (6)$$

and S_{we} now can be estimated.

When the material balance equation for hydrocarbon is multiplied by the true vertical thickness of the hydrocarbon-bearing layer, either side of this equation produces the volume of hydrocarbon per unit area at in situ conditions of temperature and pressure.

Finally, for the evaluation of R_{we} , and S_{wt} in turn, both R_w and R_{wb} must be known. In the event neither R_w nor R_{wb} is known, these values most often can be estimated by interactive computer graphics from a crossplot of R_{wa} (or C_{wa}) versus *Clayiness* (% clay) as shown in **Figure 5**, from Ransom(1995), where clayiness is estimated by appropriate clay-shale indicators. R_{wa} is determined by dividing the corrected value of R_t by F_t where $F_t = 1.0/(S_{wt}\varnothing_t)^{m_2}$ and F_t has resolved to $1.0/(\varnothing_t^{m_1})$ because S_{wt} always must be 1.0 for the determination of R_w and R_{wb} . The value for exponent m_1 should be the same value of m that will be used for the final interpretation. The value of R_t may be taken from wireline tools or measured while drilling. As pointed out earlier, the measured R_t should be corrected for environmental conditions and measuring-tool characteristics. In a figure such as **Figure 5**, R_{wa} values from zones known to be or believed to be 100% water-filled often describe a trend or curve identified as an R_{wz} trend where hydrocarbon saturation is zero. This trend can take virtually any curvature, steep, convex, concave, or flat, depending on the layering and isotropical resistivity relationship between R_w and R_{wb} . If the clay minerals within the reservoir bed are detrital, they almost always will have similar electrical properties to the surrounding clay shales. However, the clays found within a reservoir bed can be different from the clays surrounding the bed, particularly if the clays within the bed are authigenic, i.e. detrital minerals that have dissolved and re-formed in place as pore-lining crystals. If the difference in electrical properties is measurable, a dog-leg can appear in the described trend. The trend, however, is important for saturation analysis only throughout the reservoir beds. It is important that the electrical behavior and clay-indicator behavior of clays be consistent and repeatable. Once the R_{wz} trend has been established within the reservoir beds it can be extrapolated to 0% clay for R_w , and extrapolated to 100% clay for R_{wb} at in situ conditions. These values of R_w , R_{wb} , and R_{wz} have been estimated from preliminary formation factor and clay indicator information and are subject to examination by the analyst.

Although, in Figure 5, the end point of the R_{wz} trend for R_{wb} is said to be defined at 100% clayiness, 100% clayiness might not exist for

the formation. But, when Figure 5 is used as a reconnaissance tool, it is not important to know the actual amount of clay present for the estimation of water saturations. The X-axis could be renamed *Clay Index* for this work. The relationships in the vertical axis will not change if the scale on the X-axis is changed. It is important, however, that the clayiness measurement methods and resulting clayiness estimations be consistent and repeatable.

The values of R_w and R_{wb} determined from Figure 5 can be used in Eq.(1b) because they are compatible with the corrected R_t from which they came. These values are compatible because they have been derived by the same method from measurements by the same resistivity measuring device at the same environmental conditions of temperature, pressure, and invasion profile at the same moment in time.

However, the R_{wz} values, or simulated R_{we} values, found between R_w and R_{wb} are similar to those calculated from Eq.(1a). Be that as it may, Eq.(1b) becomes Eq.(1a) when $S_{we} = S_{wt} = 1.0$ at all points along the R_{wz} trend. The instant that oil or gas becomes present, S_{we} and S_{wt} become less than 1.0, and Eq.(1a) becomes Eq.(1b). Hydrocarbon saturations decrease the volume of water with resistivity R_w , and influence R_{we} by making its value move closer to the resistivity value of R_{wb} . See **Figure 2**. If R_{we} were to plot, it would produce a departure in the vertical axis from the R_{wz} trend, either higher or lower depending on whether R_{wb} is higher or lower than R_w . But, it will not plot as R_{we} from Eq.(1b) because it does not exist in nature at 100% water saturation, so there can be no natural data. R_{we} from Eq.(1b) becomes another R_{wa} the instant S_{we} and S_{wt} become less than 1.0. The R_{we} of Eq.(1b) exists only in mathematical form within a dual-water concept. If there is a significant difference between the end-point values of R_w and R_{wb} the difference between the R_{wz} and the calculated R_{we} increases. See **APPENDIX (A)** for further explanation.

It is interesting to note, however, that from **Figure 5** alone, once an acceptable R_{wz} trend has been established, that generalizes R_{we} , estimated water saturation can be previewed by

$$S_{wt}^n = R_{0 \text{ corrected}}/R_t = (F_t R_{wz}) / (F_t R_{wa}) = R_{wz}/R_{wa} \quad (4c)$$

It can be seen here that the previewed value of S_{wt}^n can be estimated independently of porosity, m , even R_w and R_{wb} , all in terms of an estimated input value for clayiness at each specific depth. The value of exponent n lies between its minimum value of m , corrected or as established above, and its maximum value determined from irreducible water saturation. Furthermore, unlike the more rigorous Eq.(4b), once exponent n has been established, S_{wt} can be previewed or estimated directly from Eq.(4c). A cautionary note appears in **APPENDIX (C)**.

Figure 5 has other uses than as a preview of water saturations in shaly sands. Figure 5 can be used as a reconnaissance tool to predict the abundance of organic matter or total organic carbon, or the occurrence of oil- or gas-deposits in shales and marlstones for further analysis. Also, it can be used as a means for selecting depths for taking additional measurements on available cuttings samples. It is one of the purposes of this plot to direct attention to zones of special interest for further investigation.

CHALLENGING WELL-LOG EXAMPLES

This part will address two vintage well log examples, chosen because of their uniqueness and simplicity. All well bores in all well examples have been drilled with a fresh-water-based mud as verified by the SP curves. **Figure 7** illustrates an oil-bearing bed in Well 1. **Figure 8** illustrates the same oil-bearing bed in Well 2. It is not known which well was drilled first. There is no information about the reservoir beds in either example other than Well 1 produced much water and very little or no oil in a production test(s). The question is: Why does the bed in Well 1 not produce oil? Observe **Figure 7** and **Figure 8** during the following discussions.

Figure 7, Description of Well 1. This well has a significant resistive marker at the depth of 2516. The zone of interest is Zone A that lies between the depths of 2545 and 2620. This zone does contain oil. Zone A was tested over the marked intervals in the depth column and produced a large quantity of water and little to no oil. This bed, Zone A, exhibits a very distinctive annulus proving that there is both movable oil and movable water in the radial zone disturbed by invading drilling-mud filtrate. The deep resistivity curve suggests that R_t is 100 ohm-meters or greater and transitions to the predominantly water-bearing Zone B that lies between 2620 and 2629. In the predominantly water-bearing zone, R_t appears to be around 14 ohm-meters. Could this be R_0 ? No other information is available. The question is: Why did a production test on Zone A recover a large amount of water from an apparently viable oil-bearing zone?

FIGURE 8, Description of Well 2. This well example is from the same stratum as Well 1 above. This well has the same marker as in Well 1 above, at 2536. The zone of interest is Zone A that lies between the depths of 2562 and 2596. This zone also contains oil. The SP currents are responding to the interference due to the presence of oil. In contrast to Well 1, Zone A in Well 2 does not exhibit an annulus. Why? The deep resistivity curve suggests R_t is around 100 ohm-meters and the bed transitions to Zone B that lies between 2596 and 2631 that appears to be wet with a resistivity of about 10 ohm-meters. Zone B transitions again to zone C that lies between 2631 and 2639 that appears to be even wetter with a resistivity of about 4 ohm-meters. Zone A at the upper part of this bed appears to

contain oil. This bed in Well 2 provides more information than the equivalent bed in Well 1, however, the question still remains: Why was the large amount of water produced from what appears to be a viable oil-producing zone?

Channeling of water from the water-bearing zones B or C is not expected to be the answer, although it is a remote, but unlikely, possibility. But, it will be seen that there is another, more plausible possibility.

Information Derived from Logs from Wells 1 and 2: The calibration of the well logs on these two wells probably is not perfect, and there probably are inconsistencies between the two wells.

In **Figure 7, Well 1**, the gamma ray tool is very sensitive and shows some statistical variation. Zone A between 2545 and 2620 exhibits a very-well-developed annulus. The presence of an annulus means that there is both movable oil and movable water within the radial zone disturbed by invading drilling-mud filtrate. The radius of mud-filtrate invasion is shallow. The radius of invasion and resulting disturbance ends at the leading edge of the annulus. The R_t for this bed probably is in excess of 100 ohm-meters. The presence of mobile oil does not mean that the oil can be produced. It does mean, however, that the bed is very porous and permeable. With no other information to guide us, it might appear that Zone B between 2620 and 2629 is nearly 100% water saturated. If so, then R_0 would be about 13 ohm-meters. The ratio of R_t/R_0 of resistivity in the upper zone to that in the lower zone is about 10:1 and surely merits further investigation. The upper zone was tested over the depth intervals marked in the depth track and much water was recovered along with an insignificant amount of oil.

In **Figure 8, Well 2**, the zone of interest is Zone A between 2562 and 2596. The interesting thing in this zone is that there is no annulus. The hydrostatic pressure is great and mud-filtrate spurt-loss invades porous and permeable rock immediately as the drill bit penetrates the rock. Impermeable mud cake cannot develop on the face of the rock until the drill bit passes by and leaves the face of the rock open. An annulus then develops along with the creation of a filter cake that soon becomes an impermeable membrane, but begins to dissipate when the mud-filtrate no longer can invade the rock. Well 1 probably was logged with the resistivity tool immediately after drilling-mud circulation ceased and drill pipe was retrieved from the bore hole. The well, therefore, was logged while the annulus was still visible. After the mud circulation stopped and drill pipe was withdrawn from Well 2, the well operation probably incurred some delay before logging began. During that lull the annulus had time to dissipate before the well was logged. This behavior is not unique. Examples of this behavior have been observed by the author in "before" and "after" well logging sessions on numerous occasions, and are described in Ransom (1995). The dissipation of the annulus by the diffusion process is common

in high-porosity, highly permeable reservoir beds, particularly where relative permeability to water is high compared with that to oil. In Zone A in Well 2, with the dissipation of the annulus and invaded zone, all resistivity curves read nearly the same and all are believed to approximate R_t . Has this zone and the upper zone in Well 1 been recently depleted and only oil at residual saturation remains? Why do these zones have such high resistivity and still produce only water?

A second zone in Well 2, Zone B, appears between 2596 and 2631. The radius of invasion is deeper in this zone than in Zone A because a lesser amount of oil is present and the relative permeability to water is higher. This zone has a resistivity of about 10 ohm-meters. This resistivity is not R_0 because there is another zone below from 2631 to 2639 that shows even a lower resistivity. This bottom zone (Zone C) has a resistivity of about 4 ohm-meters and probably is R_0 for both of the reservoir beds shown in Wells 1 and 2. If so, this makes the occurrence of all the water in the production tests that much more puzzling. Have the interesting top zones A in each well been depleted by recent emigration and now exhibit only residual oil? Then, what about Zone B between 2596 and 2631 in Well 2? This zone probably exhibits remnant oil after oil migrated out of the bed by natural means over geological time. This zone corresponds to Zone B between 2620 and 2629 in Well 1.

A depth marker in Well 1 appears at 2516. This marker could be compared with the correlative marker 2536 in Well 2 to help determine the depth relationship of one bed relative to the other and might help to explain any migration. But, the elevation of the depth datum is not known in either well so the true depth relationship between the A Zones cannot be determined. It remains that a quasi interpretation must be made in an effort to explain why neither zone of interest produces oil.

Deductive Interpretation: The potential oil-bearing beds in Wells 1 and 2 are found at shallow depths. Both porosity and effective permeability to water are expected to be high because of the lack of compactive overburden at the shallow depths of the beds of interest. This is supplemented with the reality that large amounts of water was recovered in the production test(s).

A second reason that the rock is thought to be very porous and highly permeable is because of the complete dissipation by diffusion of drilling-mud-filtrate in Zone A of Well 2. The porosity of Zone A in each well is expected to be higher than 25%, perhaps as high as 30%. Porosity of the bed and relative permeability to water are high and relative permeability to oil is low in this reservoir bed. Because of the expected good quality of the rock, the value of exponent m is believed to be low.

Resistivity R_0 can be taken from zone C in Well 2, and is believed to be about 4 ohm-meters. Resistivity R_t in Zone A in Well 1 is in

excess of 100 ohm-meters and in Well 2 is approximately 100 ohm-meters. It is possible that the difference in calibration between the two logging tools would account for some of the difference in R_t values, but the more likely reason is the differences in R_{we} and water-filled ϕ_{ne} .

It is believed that oil might be at residual saturation in each Zone A, by emigration or by prior production from nearby wells. But what is the residual oil saturation value? It is not known, but could be 20%, 30%, 40%, 50%, maybe more.

The assumptions for any further calculations are: $\phi_t = 0.30$, $\phi_{ne} = 0.05$, and $m = 1.6$. It should be noted that experienced analysts will agree that these assumptions are most favorable for the calculation and prediction of oil production.

The calculated R_w from zone C in Well 2 is about: 0.58

Questions inspired by these wells are:

- a) Should the prominent oil-bearing bed in either well be production tested?
- b) Should casing be set in either well?
- c) Is water channeling to the test depth from a location lower in the bed?
- d) Should attempts be made to isolate the tested depth interval in Well 1 and re-test?
- e) Should Zone A in Well 2 be tested?
- f) Can the water saturation be estimated?

Questions elicited here can be addressed with a certain amount of caution:

- a) To be absolutely certain that primary oil production is not viable, Well 1 probably should have been tested, as it was, but with the expectation that no oil would be recovered.
- b) No oil can be produced by primary production methods in either Zone A. No casing should be set in either well, unless enhanced oil recovery methods are entertained. Any method of recovery that can decrease the viscosity of the oil should be considered. If the viscosity of oil in Zone A can be reduced, whatever oil would be produced would have a very high water cut and the operator producing such oil would have to be prepared to handle the water produced.
- c) Water is not channeling from a lower level to Zones A or B in

either well.

d) No attempt should be made to isolate the test interval in Well 1 and re-test.

e) Zone A in Well 2 should not be tested.

f) What is the residual oil saturation in Zone A or B in either well? Because of the high resistivity and ample thickness in both Zones A, it is possible that enhanced oil recovery methods could be successful, but might not be economical. If considered viable and employed, perforations should be set high in the bed and withdrawal rate should be low to minimize coning by water. But, as it will be shown below, only the maximum calculated oil saturation can be predicted from these well logs and Figure 10. The actual oil saturation in any zone is speculative. It cannot be determined from these resistivity well logs alone. Other well logs and/or other disciplines should be employed.

A general conclusion is the reservoir bed is oil-wet because R_t in Zones A for both Well 1 and Well 2 have such high values in a rock where the assumed porosity is so high. Such resistivity values for R_t are 100 ohm-meters or more in the suspected oil-bearing zone (and can be much higher in other oil-wet rocks) but the probable R_0 is only about 4 ohm-meters.

The relative permeability to water is much higher than the relative permeability to oil. This is corroborated by the large amount of water recovered in production tests.

Because the relative permeability to water and that to oil is significantly different, and because the mobility of water (k_w/μ_w) is greater than the mobility of oil (k_o/μ_o) an annulus can form in Zone A of Well 1. Because the mobility of water is high, the time required for an annulus to develop or to dissipate is short as it was proposed for Well 2

The reason why large amounts of water along with virtually no oil was produced during production test(s) in Zone A, Well 1, is believed to be because the mobility (k/μ) of water is so high and the mobility of oil is so low.

Because of the logic immediately above, the oil is expected to be very viscous, and the presence of oil is likely to reside at residual saturations in Zones A and B in both wells.

Because the proposed residual oil saturations in Zones B are lower than the oil saturations in Zones A, it is believed that the oil originally in place in both Zones B has been reduced by natural emigration over geologic time.

OBSERVATIONS AND CONCLUSIONS FROM FIGURE 10 ABOUT EXPONENT n

Examine the curves drawn in **Figure 10**.

All calculations of exponent n shown in **Figure 10** are based on triangle CDG of the model in **Figure 1**. Triangle CDG is the graphic depiction of the dual-water Archie two-exponent water saturation Eq. (4b).

The values of the calculated n from the logic of triangle CDG in the model and from Eq. (4b) are summarized in the type curves drawn in **Figure 10**.

How to Interpret Figure 10. Caution. Do not think of the curves in Figure 10 as profiles.

Each type curve represents a relationship between exponent n and saturation S_{wt} in a specific reservoir bed or same sample of rock at a deep radial depth beyond the invaded zone.

Imagine each of the curves as a series of smoothed bar graphs for exponent n over a full range of water saturations at a constant value of R_t . At any specific water saturation, the n value at that water saturation represents the degree of, or the effectiveness of, the interference by oil to the flow of electrical survey current in oil-wet- or water-wet rocks. It can be seen in Figure 10, except at very high water saturations, that the lower the value of n , the lower is the cumulative interference by oil or gas, and the higher is the oil or gas saturation for the prevailing resistivity.

Not all of the chart is applicable. The values of exponent n are meaningful only at oil saturations greater than irreducible oil saturation. The values of n can be calculated in the region of irreducible oil and in the flushed zone only if the appropriate value of R_t is known for that saturation, and employed.

The curves are applicable only where a constant value of R_t remains valid. The curves are not valid where water saturations are high and exponent n is steep because a high value of R_t cannot be supported at high water saturations. At high values of water saturation, exponent n will begin a decline and continue to decline as water saturations increase until it reaches the value of exponent m . In Figure 1, resistivity R_t also will decline along with the decline of n until it reaches the value of R_0 .

As a result, the steep upward mathematical sweep of n values probably does not exist in nature. The high values of n can be calculated, but at high values of water saturation R_t must ultimately return to R_0 , thus n will return to m .

One of the first things to be observed in **Figure 10** is that in both

Zones A and B in Wells 1 and 2, as the resistivity gradient described by exponent n increases, predicted water saturation increases and the predicted oil saturation decreases. And, where the oil saturation is the greatest, exponent n is the lowest. Puzzling? This is in precise conformity with the prediction by exponent n illustrated by slope CG of triangle CDG in **Figure 1**.

For example:

1. If the voids in the rock are filled to a high oil saturation, an increase of 10% oil will decrease the volume of electrically conductive water correspondingly.
2. If the voids in the rock are filled to a low oil saturation, an increase of 10% oil will displace the same bulk volume of water as at high saturations.
3. Wherever the oil phase is continuous, whether at low saturations or high saturations, the electrical interference caused by adhesive insulating oil films on the pore surfaces will be nearly the same.
4. Although the electrical interference by oil in items 1 through 3, immediately above, is the same wherever the oil exists in a continuous phase, Figure 10 and the model in Figure 1 illustrate that the effectiveness is not the same.
5. The effectiveness of the interference by oil is greater at higher water saturations than at lower water saturations as seen by the increased values of n . This phenomenon is corroborated by the model in Figure 1, Equation (4b), and the type curves in Figure 10.

In Figure 10 what do the curve crossovers mean at point H? Figure 10 illustrates that everywhere oil is present, the true value of exponent n is greater than exponent m , but some calculated values of n are lower than m . Why?

The calculated oil saturation is highest where exponent n approaches exponent m . In both **Figure 1** and **Figure 10** that maximum calculated oil saturation level is seen at point H. In Zones A in Wells 1 and 2, that maximum occurs at about 90% oil saturation (disregarding irreducible water saturation) where the curves for Zones A meet the assumed value of 1.6 for exponent m .

In **Figure 10**, because the n curves for Well 1 Zone A and Well 2 Zone A, intercept the value of m at high oil saturations, it would imply that there is more oil to be produced from this reservoir. For Zone B, the highest oil saturation that can be calculated is about 50% where the curves for zones B also meet the value of exponent m at a point H. This behavior is illustrated in **Figure 1** where line CG (slope n) of triangle CDG collapses to line CH that represents slope m . Any calculated value of n lower than the value

of m is imaginary.

What about the imaginary values of n in Figure 10? The oil saturation where the calculated n crosses the value of exponent m , at locations H, represents the highest oil saturation that can be calculated for that data and that well depth. It does not imply that the specific saturation at H is the predicted saturation for water or oil. Examine the model in **Figure 1**. Remember: Exponent m becomes exponent n after some of the interstitial water has been displaced by oil. So, in **Figure 1** when gradient CG collapses along angle δ to lie on gradient m (now $n = m$) it cannot collapse any further. The maximum calculated oil saturation is predicted by the intersection of the extrapolated slope of m and the level of R_t wherever intersection H resides. This is the same H as appears on the effectiveness chart in **Figure 10**. The calculated values of n lying below the value of m do not exist. The imaginary values absolutely can be calculated, but cannot be reproduced in the laboratory or in nature. Such values published in petrophysics literature are disputable.

The values of n along the steep slope of the curves in **Figure 10**, and the values of n lying below slope m in Figures 1 and 10, is mathematics. But, the values for n falling only to the limiting value of slope m , *from whence they came*, is physics.

Exponent n is higher in Zone A than in Zone B at the same water saturation in both wells. It can be seen in **Figure 10** there is no mathematical relationship between exponent n and S_{wt} observed between one bed and another at any constant saturation value.

As R_t changes, the slope representing values for n might or might not change. Exponent n can vary independently of water saturation. For the value of n to increase above that of m , it is necessary only for oil to be present. Water saturation influences n , but, except for coincidences, there is no curvilinear relationship between S_{wt} and n except within the same bed, and the bed exhibits uniform distributions of oil, water, porosity, and other factors influencing electrical conductivity, as the individual curves in Figure 10 illustrate.

What if the imaginary values are used in interpretation? Slope m in **Figure 1** is a resistivity gradient representing the value of R_0 at 100% electrical efficiency in the selected rock at all porosities. If slope n were to lie below slope m , resistivity R_t would lie below the extrapolated values of R_0 , and oil saturations would be artificially overestimated over all porosities and saturations. This is a commonplace occurrence in literature that has become a factoid of conventional wisdom. The presence of oil cannot increase the electrical conductivity of water more than 100% efficiency at 100% water saturation (at m). See **Figure 6** and **APPENDIX(D)**.

Figure 9. Description of Well 3. This well log, too, is a vintage resistivity log. This log was chosen because of its simplicity. In this figure is a resistivity log in a water-wet rock. Like the two examples above, the resistivity curves show great contrast between an oil-bearing zone and a water-bearing zone. This example shows a dual induction log with an SP curve. At the top of the bed between X151 and X161 is an oil-bearing interval marked Zone A. Near the bottom of the bed is a water-bearing interval between X183 and X192 marked Zone B.

The calculated values of n relative to water saturations for this bed also are shown as a curve in **Figure 10**.

A moderately high value of saturation exponent n up to perhaps 8 or 9 might not be a factor in distinguishing between water-wet and oil-wet rocks.

As it can be seen in **Figure 10**, as water saturation in Well 3 increases, the calculated curve for n will increase again the same as the curves for Wells 1 and 2. This, again, is corroborated by the slope CG representing exponent n of triangle CDG in the model in **Figure 1**.

Examine the effectiveness curve from Zone A of Well 3 in **Figure 10**. This curve is plotted from values of exponent n in water-wet rock. It can be seen that the curve is very similar to the curves from oil-wet rock. Why is this so?

It is a simple matter for a computer to solve the necessary equations and iterations required to compute the results appearing in Figure 10. Let's examine how exponent n is calculated:

In the calculation of each exponent n , R_{we} first must be calculated from Equation(1b) for each iterated value of S_{wt} . Along with R_{we} , exponent n must be solved from triangle CDG in Figure 1 or from the dual-water Archie equation, Eq.(4b):

$$S_{wt}^n = (1.0/(\phi_t^{m1})) (R_{we}/R_{t \text{ measured}}) = (R_0/R_{t \text{ measured}}) \quad (4b)$$

where from triangle CDG of Figure 1,

$$n = (\log R_t - \log R_0) / (\log 1 - \log S_{wt}) = (\log R_0 - \log R_t) / (\log S_{wt})$$

It can be seen in Eq.(4b) above, that the ratio $(R_0/R_{t \text{ measured}})$ varies little for each Zone A in the Wells 1, 2, and 3. Therefore, the equation for each curve in Figure 10 is:

S_{wt}^n = a near constant for each depth of interest depending on the value of the ratio.

Solve for n by iterating S_{wt} in Eqs.(1b) and (4b).

Can oil-wet zones be distinguished from water-wet zones by high values of n ? Sometimes and sometimes not. In Well 1, Zone A, an oil-wet zone, the ratio of $(R_0/R_{t \text{ measured}})$ is approximately $10/100 = 1/10$ or even smaller. In Well 2, Zone A, also an oil-wet zone, the ratio again is approximately $10/100 = 1/10$ or thereabout (smaller if R_0 is taken directly from Zone C). And in Well 3, Zone A, a water-wet zone, the ratio is about $1/10$. So, it doesn't matter whether the zone is oil wet or water wet the calculated values of n in these cases can be nearly the same for the same water saturations. When Equation(4b) is solved for exponent n it can be seen in these ratios that there is little difference in Equation (4b) whether the rock is oil wet or water wet. Although, at high n values, the rock probably is oil wet, but it is not always the case.

In some oil-wet rocks, the ratio might be very small. The ratio of $R_0/R_{t \text{ measured}}$ might be $10/1,000$ or $10/10,000$. It can be seen in Equation(4b) that as R_t increases (or R_{we} decreases), exponent n increases. The ratio R_0/R_t in this order of magnitude moves the type curve shapes upward in **Figure 10** because exponent n increases everywhere. As R_t increases, point H on both Figure 1 and Figure 10 will be shifted to the right toward higher calculated maximum oil saturations.

The values for exponent n are viable only at higher oil saturations than irreducible oil saturations. This must be taken into consideration. The calculated bulk volume of oil consists of producible oil and irreducible oil; and the calculated bulk volume of water consists of water that can be produced, irreducible water, and water (or pseudo water) contained in dynamically non-effective rock constituents.

What about the steep upward sweep of n at high water saturations? In Figure 10, like the values of n found below the value of exponent m , the extreme values in the steep upward sweep of the curves also are imaginary. They can be calculated, as reported above and seen in Figure 10. They actually would exist if the high values of R_t actually existed at the high water saturations, but they do not. At high values of S_{wt} the bulk volume of water is electrically more conductive, and high values of R_t used in the calculations are not co-existent and do not exist in real life. Hence, the upward sweep in n at these saturations is artificial. The n values are imaginary.

The actual values of n must gradually return to the value of m at high water saturations so that at 100% water saturation and 0% oil saturation exponent n becomes m again. From some location on the effectiveness curve, as water saturation increases, the value of n will increase to a maximum whereupon, as water saturation continues to increase, the value of n declines followed by a decline of R_t until the water saturation becomes 100%, n becomes m , and R_t becomes R_0 .

The dashed curve in Figure 10 is one of a number of artful conceptions

that exponent n might take as oil migrates into or out of water-wet- or oil-wet rock, simultaneously changing R_t . As R_t increases, the ratio R_0/R_t (in Eq. (4b)) decreases and exponent n increases. As n increases, the apex becomes narrower and sharper from which n declines approaching and ultimately returning to exponent m at 100% water saturation.

In Figure 10, the effectiveness curves under these conditions might resemble normal frequency curves skewed heavily toward high water saturations, and slope CG in triangle CDG in Figure 1 might correspondingly morph into a shape similar to a lazy S as high water saturations approach 100%.

Both the model in Figure 1 and the curves in Figure 10 would be modified as R_t changes to correspond with the reasoning set forth above.

What about exponent m_2 ? This is a single saturation exponent equivalent to the two exponents m and n for saturation calculations. It is derived from triangle AEG in the model in **Figure 1**. If the effectiveness curve for m_2 were shown in the chart in **Figure 10**, the curve would lie between the constant exponent value of m and the curves for n . A close inspection of Figure 1 will show that the slope representing m_2 is greater than that of slope m but smaller than that of slope n . It also can be seen that triangle AEG has its origin at R_{we} instead of R_0 , triangle AEG is more tolerant of errors. Because of the greater length between AG over CG, slope AG will change less than slope DG for the same error in porosity or saturation.

Everything that has been said about exponent n applies to exponent m_2 , with the exception that the effects on or by m_2 will be smaller. On both Figure 1 and Figure 10, where the slope of exponent n meets the slope of m at point H, so does the slope of exponent m_2 meet slope m at point H.

ARE THERE LIMITATIONS TO ARCHIE'S RELATIONSHIPS DEVELOPED IN THIS MODEL?

Are there rocks that complicate resistivity interpretations? Of course. As in every discipline and in every analytical method, there can be problems that complicate the interpretation process. That is not unique to Archie's relationships, whether new or old. Above, it was mentioned that Archie's relationships, as they have been developed in this paper, apply to heterogeneous rocks with uniform distributions of porosity, saturations, and electrically-conductive constituents. That is a requirement for all matter subject to electrical investigations. Problem rocks are those that exhibit nonuniform distributions of causative factors that influence resistivity. Such factors or conditions can be, but are not limited to: anisotropy, beds considerably thinner than the vertical resolution of the logging

tools, bedding planes at high angles relative to the borehole axis, human error in running the wrong resistivity-measuring tool for the existing resistivity profile in the mud-filtrate invaded zone, etc. Are these Archie problems? No. These are problems that affect all related analytical resistivity methods. These are problems that either nature or the shortcomings of man have dealt to the analyst. These conditions are part of the interpretation process and require the attention of qualified well-log analysts.

CONCLUSIONS

The most important thing to be drawn from this paper is the model in Figure 1. From that model, two versions of Archie's saturation equation are derived for dual-water dual-porosity conditions: the common two-exponent version and an equivalent single-exponent version. Also, each of Archie's parameters have been derived, identified, and explained. These fundamental relationships and parameters have been derived from trigonometrics of the graphic model and corroborating algebraics, and have shown that Archie's relationships developed herein are a dual-water methodology.

Second. is the definitive exploration of the exponent n illustrating its behavior in both water-wet and oil-wet rocks. This exhaustive exploration reveals information about the saturation exponent n that never before has been revealed in literature.

The concept described herein is based on the very fundamental electrical law relative to the conversion of resistance to resistivity, and on the efficiency of the network of pores and pathways in the rock through which the electrical-survey current must flow. The basic electrical resistance equation is the key to understanding the Formation Resistivity Factor. The true formation factor equation alone contains all terms necessary for the calculation of Dual Water Archie.

Archie's relationships, as the parameters have been defined in this model, have been shown to apply in dual-water dual-porosity conditions and in shaly sands and other rocks of limited heterogeneity. The methodology developed herein can serve as the spine to which numerous correctional and mineral-specific subroutines can be attached to add analytical refinements. The model and the foregoing developments in this paper have shown that Archie's relationships have a straightforward, physical and mathematical basis, and, in this concept, have been shown to apply to heterogeneous rocks with uniform distributions of porosity, saturations, and electrically-conductive constituents.

Archie's relationships are fundamental and the foregoing developments are deemed necessary for the relationships to be understood, used in literature, and employed in saturation calculations. An understanding

of what influences these parameters and what they represent will provide the user with a place to start and a recognition of what is required from other disciplines for the solution of interpretation problems, and will complement the user's inventory of tools for describing rocks as they reside in nature.

EPILOGUE

A good well-logging interpretation program is like a double-edged sword. It must cut both ways. The program must find hydrocarbons where they exist and must not imply their existence where they do not exist. The purpose is to be accurate with consistent reliability. Together with interactive computer graphics, the author has used the foregoing logic and fundamentals as the skeletal framework to guide resistivity interpretations for many years in all parts of the world.

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SYMBOLS DEFINED

- a Historically, the coefficient of the Formation Resistivity Factor. In this paper, the coefficient of R_w . Usually equal to 1.0 in clean rocks, and different from 1.0 in the presence of any conductive influence other than formation water. Often lower than 1.0. When incorporated in comprehensive R_{we} relationships, the numerator in the Formation Resistivity Factor must always be 1.0.
- m The porosity exponent in the Formation Resistivity Factor. Related to the shape and dimensions of electrolyte-filled paths. There are two versions of exponent m . One is the usual m (or m_1) for the two-exponent version of Archie's equation, and the other is m_2 for the equivalent single-exponent version.
- n The saturation exponent found in Archie's water saturation equation. Related to the electrical interference caused by the presence of hydrocarbon in electrolyte-filled paths. What exponent m becomes after hydrocarbon has migrated into the rock and has displaced water.
- \emptyset_t Total porosity fraction consisting of the sum of effective and noneffective porosity fractions.

- \emptyset_e That part of the total porosity fraction that is hydrodynamically effective and can be occupied by hydrocarbon.
- \emptyset_{ne} Is the noneffective porosity in clay shale related to water attracted to the clay crystal and water held osmotically, and is directly related to the fraction of clay shale present. It is a porosity volume that has not been penetrated by hydrocarbon. Water saturation in the noneffective pore space is 1.0 and resistivity is represented by R_{wb} .
- F_t The Formation Resistivity Factor of the rock, where the conductive water volume fraction always is $S_{wt}\emptyset_t$, where $S_{wt} \leq 1.0$. In this paper, the numerator always is equal to 1.0.
- R_w Formation water resistivity. Interstitial-water resistivity.
- R_{wa} The apparent resistivity of formation water found by dividing R_t by F_t , where $S_{wt} = 1.0$ and $m_2 = m$.
- R_{wb} The apparent resistivity of water bound in clay shales.
- R_{we} Same as R_{weq} in other work. A calculated value for the equivalent water resistivity when secondary conductive influences are present. In this paper, the product of coefficient a and R_w as it is incorporated in Equation (1b) for the calculation of R_{we} . Water with resistivity R_w is interstitial water. Water with resistivity R_{we} (or R_{weq}) is not interstitial water and does not exist in nature at 100% water saturation. It is a hypothetical water mixture of dual waters, influenced by the presence of oil, that exists at 100% water saturation only in a dual-water concept. Electrically the dual waters can be mixed, physically they cannot. This dual-water mixture cannot be recovered in a production test.
- R_{wz} Is the R_{wa} value determined from a zone known to be or believed to be completely filled with water or nearly saturated with water. Hydrocarbon saturation is zero or near zero. R_{wz} is a graphical approximation of R_{we} and a special condition of R_{wa} .
- R_0 The true resistivity of the uninvaded rock when water saturation is 1.0. The water or pseudo water can have the resistivity value R_{we} . Sometimes calculated, sometimes read directly from the deep resistivity curve, sometimes derived from the deep resistivity curve. R_0 is a special condition of R_t .
- R_t The true resistivity of the uninvaded rock, whether or not it contains hydrocarbon. Sometimes read directly from the deep resistivity curve, sometimes derived from the deep resistivity curve, sometimes calculated. R_0 is a special condition of R_t .
- S_{we} Water saturation as a fraction of the effective porosity \emptyset_e .

S_{wt} Water saturation as a fraction of the total porosity ϕ_t .

$S_{wt}\phi_t$ The total conductive water volume fraction where $S_{wt} \leq 1.0$. A fraction of one unit of formation volume. Sometimes referred to as the bulk volume of water.

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APPENDIX

PREFACE. The model appearing in **Figure 1** incorporates **Figure 2**. The origin always is R_{we} , the apparent or equivalent resistivity of formation water. But, R_{we} is the same as R_w in clean rocks and different from R_w in heterogeneous rocks, and in the presence of hydrocarbon saturation. The influence of hydrocarbon saturation on R_{we} can be observed in Equation (1b) found under **PARALLEL RESISTIVITY EQUATIONS USED . . .**

The Y-axis is R_t , the resistivity of rock containing water. The X-axis is $S_{wt}\phi_t$, the bulk volume of water, consisting of two parts, ϕ_t and S_{wt} . It must be recognized that ϕ_t and S_{wt} have no units, but are fractions and in themselves are not volumes. As fractions, neither ϕ_t nor S_{wt} has resistivity because neither is dimensional. Only bulk volume water can have volume and, therefore, can have resistivity. This could be confusing, because the fraction that is bulk volume water in literature must be multiplied by a dimensional conversion factor that has a value of 1.0 and appropriate three-dimensional units. Now being dimensional, the bulk volume water can have resistivity.

In the discussion of the Y-axis, the basic resistance-resistivity equation must be introduced. That equation is:

$$\text{resistance}_{\text{water}} = (L/A) \times \text{resistivity}_{\text{water}}$$

$$\text{Dimensionally,} \quad \begin{array}{l} \text{ohms} \\ \text{(water)} \end{array} = \begin{array}{l} \text{meters/meters}^2 \\ \text{(F)} \end{array} \times \begin{array}{l} \text{ohm meters}^2/\text{meter} \\ \text{(water)} \end{array}$$

The expression (L/A) is a formation resistivity factor in its simplest form. It converts resistivity to resistance, and resistance to resistivity. So, how does resistance of water become resistivity of rock?

In **Figure 1**, R_0 and R_t are resistances of water per unit length, and at the same time resistivities of the unit volume of rock that contains that water. In inert, electrically non-conductive rock, water

is contained in the pore spaces of the rock. The water volume occupies pore paths and creates electrically conductive paths that exhibit various levels of electrically conductive efficiency. These paths can have various degrees of resistance as the dimensions of (L/A) are varied in the resistance-resistivity equation above. However, the resistance of the water over the length of rock in a unit volume of insulating rock constitutes the resistivity exhibited by that rock. Numerically, the resistance of water is the same as the resistivity of rock,

In **Figure 1**, the resistance of water over a unit volume of rock is the resistivity of the rock. The Y-axis in Figure 1 is resistivity of rock except at 100% porosity where it is the resistivity of water.

(A) Figure 2. This figure shows the function of coefficient a . In the section above related to coefficient a an equation was written showing all the variables used in the calculation of a . One of the variables was water saturation in the effective pore space. Why is that necessary?

The presence of hydrocarbon decreases the volume of water in the effective pore space. This changes the proportionality of water volume with resistivity R_w relative to the volume of pseudo water with resistivity R_{wb} that is associated with clay shales. In a parallel conductivity relationship this moves R_{we} closer to the value of R_{wb} as can be seen in Figure 2.

Input value R_w is from the best known source or from **Figure 5** after clayiness has been determined by the best clay indicators available. Porosity values, ϕ_e and ϕ_t , are determined by traditional methods, or from Ransom (1977, 1995), after employing the appropriate matrix values for the host rock.

Coefficient a is shown to be a multiplier of R_w but is never calculated independently for use in an interpretation program when it is an integral part of R_{we} , as seen in Eqs. (1a) and (1b). In dual-water dual-porosity methods, coefficient a may be calculated only as a matter of interest, but must never appear in a saturation equation with R_{we} .

(B) Figure 1. Figure 1 illustrates the model that Archie's relationships obey. It is intended for informative and illustrative use, only. It is not drawn to scale. The inclinations of the straight lines representing slopes are determined by trigonometric tangents. The value of a specific trigonometric tangent is the value of the specific m or n . The steeper is the slope, the greater is the interference and resistance to electrical-current flow through the pores and pore paths in the rock, the greater is the inefficiency for the transmission of electrical-survey current, and the greater will be the value of m or n .

(1) On the X-axis, it is seen that $S_{wt}\phi_t$ decreases to the right. A study of the logarithmic scales will show, for example, that $\phi_t = 0.2$ and $S_{wt} = 0.3$; and, as a result, their product $S_{wt}\phi_t = 0.06$.

(2) There are three right triangles of interest in **Figure 1**. They are triangles ABC, CDG, and AEG.

In any triangle shown in the figure, the slope or tangent of an angle is described as the side opposite divided by the side adjacent. The trigonometric law pertaining to tangents, that right triangles follow, can be described as:

$$\text{tangent of acute angle} = (\text{side opposite})/(\text{side adjacent})$$

Because the X-axis is descending in value to the right of the origin, the sign of the tangent will be negative. Therefore, on the log-log plot such as **Figure 1**,

$$(\text{tangent of acute angle})(\log(\text{side adjacent})) = \log(\text{side opposite})$$

and, $1/(\text{side adjacent})^{\text{tangent of acute angle}} = (\text{side opposite})$

Trigonometry was created for solving problems. Express these trigonometric functions in equation form and the Formation Resistivity Factor and an improved Archie's water saturation relationship will emerge.

In **Figure 1**, the tangent of the acute angle β of the right triangle AEG is represented by

$$\begin{aligned} \tan \beta &= m_2 = {}_{\log} (EG) / (\log 1 - \log(AE)) \\ -m_2(\log(AE)) &= \log(EG) = \log F_t \\ F_t &= 1.0 / (AE)^{m_2} \end{aligned}$$

In equation form, this is

$$\begin{aligned} m_2 &= \log F_t / (\log 1 - \log(S_{wt}\phi_t)) \\ -m_2(\log(S_{wt}\phi_t)) &= \log F_t \\ F_t &= 1.0 / (S_{wt}\phi_t)^{m_2} \quad . . . (3b) \end{aligned}$$

In **Figure 1**, the tangent of the acute angle γ of the right triangle CDG is represented by

$$\begin{aligned} \tan \gamma &= n = \log(DG) / (\log 1 - \log(CD)) \\ -n(\log(CD)) &= \log(DG) = (\log R_t - \log R_{0 \text{ corrected}}) \end{aligned}$$

$$(CD)^n = R_{0 \text{ corrected}}/R_t$$

In equation form, this is

$$\begin{aligned} n &= (\log R_t - \log R_{0 \text{ corrected}}) / (\log 1 - \log S_{wt}) \\ -n(\log S_{wt}) &= (\log R_t - \log R_{0 \text{ corrected}}) \\ (S_{wt})^n &= R_{0 \text{ corrected}}/R_t \quad . . . (4b) \end{aligned}$$

(3) $(S_{wt}\emptyset_t)^{m_2}$ has the same function as and is equal to $(S_{wt})^n(\emptyset_t)^{m_1}$.

In triangle AEG, $-m_2(\log(S_{wt}\emptyset_t)) = \log R_t - \log R_{we}$

$$(S_{wt}\emptyset_t)^{m_2} = R_{we}/R_t$$

In triangle CDG, $-n(\log S_{wt}) = \log R_t - \log R_{0 \text{ corrected}}$

In triangle ABC, $-m_1(\log \emptyset_t) = \log R_{0 \text{ corrected}} - \log R_{we}$

adding the two equations that involve n and m_1 , yields

$$-n(\log S_{wt}) + (-m_1(\log \emptyset_t)) = \log R_t - \log R_{we}$$

$$S_{wt}^n \emptyset_t^{m_1} = R_{we} / R_t$$

But, $(S_{wt}\emptyset_t)^{m_2} = R_{we}/R_t$, from triangle AEG above.

Therefore, $(S_{wt}\emptyset_t)^{m_2} = (S_{wt})^n(\emptyset_t)^{m_1} \quad . . . \text{ same as (3c)}$

This equivalence also was proved graphically in Eq.(8) on page 7 in Ransom (1974).

(4) In addition, in partially oil-wet or oil-wet rocks, the assumption that $n = m$ can lead to calculated water saturations that are too low. In **Figure 1**, it can be seen that when the default value of n equals m the line representing n will intercept the R_t level far to the right at H. The resulting calculated water saturation can be too low and might even be "unreasonable". When the corrected n value is used, the slope representing n will intercept the level of R_t at point G somewhere near point H or to the left of H, which will yield a "reasonable" value for S_{wt} depending on the value of n . Straight lines representing values of n can rotate along the arc δ as either n or S_{wt} varies. The slopes representing n should intercept the corrected value of R_t at calculated water saturations within the saturation range between irreducible water and irreducible hydrocarbon.

(5) In oil-wet or partially oil-wet rocks the presence of oil will increase the interference to the flow of electrical current, causing the value of the saturation exponent n to increase. In **Figure 1**, it can be seen that as the value of the saturation exponent n increases,

the slope of n increases. In Figure 1, under the condition that there will be no increase in R_t , as the slope of n increases, the calculated water saturation will be seen to increase. In oil-wet rock, a film of viscous adhesive oil coats the walls of the pores and pore throats. The value of n is related to the degree of electrical interference caused by the adhesive oil, and that degree of interference can be caused by adhesive oil films whether or not the oil saturation and oil distribution is sufficient for oil production to take place. **Figure 1** shows that the saturation exponent n is the resistivity gradient that R_t employs between $R_{0 \text{ corrected}}$ and R_t relative to changes in the saturation distribution of oil, not to the value of oil saturation. It is related to the value of oil saturation only through R_t . The saturation exponent n is not to be confused with the saturation value of oil. Figure 1 suggests that with no increase in R_t and with varying degrees of effectiveness of the coating of pore walls and pore throats with adhesive, viscous oil, this condition can allow a high water saturation to exist with the result that no oil will be produced, or oil will be produced with a high water cut. If oil is to be produced with a low water cut or with zero water cut then the oil must reside in a continuous phase that allows the oil to move and in such quantity and distribution that it causes discontinuity in the water phase. If the presence of oil resides in a continuous phase in oil-wet rock, this condition will present further increased interference to the flow of electrical survey current with the result that the value of R_t will increase to uncommonly high resistivity levels.

It might be inferred from Figure 1 that the ideal condition for oil production to take place in oil-wet rocks would be for the value of n to be as low as possible and the value of R_t to be high. Under this combination the slope of n will intercept the level of R_t farther to the right on the X-axis. Calculated water saturation will be lower and oil might be produced with a reduced water cut or no water cut. Where are these conditions likely to be found? Examples might be in rock containing relatively large, well sorted grains or in rock with well interconnected dissolution porosity. These conditions will be found in good quality reservoir rock where the value of porosity exponent m is low.

(C) Figure 5. This figure is a plot of R_{wa} vs *Clayiness* (% clay). It could have been a plot of corrected R_t vs *Clayiness*. The plot of R_{wa} vs *Clayiness* yields more information relative to secondary constituents. The plot also has uses in focusing attention on data and depths where unexpected events or mineralization occurs.

When calculating clayiness for this plot, clayiness should be determined from the best clay-shale-responsive indicators available, but never from an average of several. Consider the influences on each of the clay-responsive measuring methods before each is entered into the clay-estimation process.

(D) **Figure 6** is nearly the same as Figure 1. Like Figure 1, Figure 6 is not intended to be a working graphical procedure, it has been exaggerated to illustrate detail, and it is intended only to be informative and explanatory. In **Figure 6** it is shown that the same rock can contain the same volume fraction of water, $S_{wt}\phi_t$, whether or not oil or gas is present. In Figure 6, the example uses the same porosity and saturation values as seen depicted in Figure 1. In this example, m is uniform throughout the host rock. If water saturation is 100% when the porosity is reduced to 0.06, then $S_{wt}\phi_t = 1.0 \times 0.06 = 0.06$. This is the value of the total water volume as a fraction at J. The same volume of water can be observed in the same rock when the porosity is 20%, but oil or gas displaces 70% of the water volume: $S_{wt}\phi_t = 0.30 \times 0.20 = 0.06$. This is the total water volume as a fraction at K. This common value of $S_{wt}\phi_t$ is shown at E at the base of Figure 6 on the logarithm scale for $S_{wt}\phi_t$.

In **Figure 6**, where the example rock shows a water volume fraction of 0.06, it can be seen that when $n < m$, R_0 at resistivity level J will be greater than R_t at resistivity level K, for all values of porosity and all values of saturation. In real life this cannot happen. But, when the value of n is found to be or made to be $< m$ in the same sample of rock, most users will not recognize the impossibility of this condition.

When n is said to have a lower value than the host value of m , the associated values of R_t always becomes less than R_0 at equivalent water volumes in the same rock. And that associated value is profoundly incorrect. For R_t at K to have a lower value than R_0 at J, the migration of oil or gas into the interstitial water volume must make the electrical paths through the remaining water volume more efficient. This enhancement of the electrical conductivity of the water paths through the interstices by the presence of oil, must be done with no change in salinity. Hypothetically, for oil or gas to migrate into the undisturbed effective pore space and cause the value of n to be $< m$, the presence of oil or gas must cause the electrically conductive paths of water to become more efficient. This, the presence of a continuous phase of oil or gas in water-wet rock cannot do. In the model in this paper, which is faithful to laws of physics, when all other things remain unchanged, it is shown that it is impossible for the actual value of R_t to have a lower value than the extrapolated value of R_0 for the same quantity of water in the same sample. But, can there be an exception to this rule? Probably not. To do so would be in conflict with physics. Any occupation of pore space by oil decreases the electrically conductive volume of water and, therefore, increases the resistivity gradient.

When $n < m$ the slope representing the n becomes so low that for whatever value of R_t exists, the slope representing an aberrant n will intercept that level of R_t far to the right at an artificially low water saturation value on the right-facing Figures 1 and 6. The consequence is that the calculated water saturation will be too low

and the hydrocarbon saturation will be over estimated indiscriminately in producible and non-producible hydrocarbon-bearing beds alike. This will be an insidious by-product that most users will not anticipate.

To accept a value for n that is lower than m , is to deny that the migration of oil or gas into rock will cause more electrical interference than water alone. That denial contradicts basic physics and violates the fundamental principles of resistivity analysis.

How the presence of oil can make water more electrically conductive will not be explained in this paper. For explanations why values of n lower than values of m have been measured, and why the results have been overwhelmingly embraced in literature, the reader will have to look elsewhere.

ABOUT THE AUTHOR



Robert C. Ransom is a retired consultant in formation evaluation through the use of well-log analysis. Formerly he was Senior Research Associate in Formation Evaluation for UNOCAL Corporation's research center at Brea, California. He graduated in 1951 with a B. E. Degree in Chemical Engineering from the School of Engineering at Yale University. He is an Honorary Member of the Society of Petrophysicists and Well Log Analysts (SPWLA). He joined Schlumberger Well Surveying Corp. in 1951 and was on leave of absence to serve with the Chemical Corps of the U.S. Army during the Korean War. He left Schlumberger to research and develop well-log analysis methods at UNOCAL's research center and to work with the field department on problems of practical importance. Bob compiled and edited both the 1st and 2nd editions of the SPWLA Glossary. He has authored and published a number of technical papers, a textbook, and an Interpretive Groundwater Glossary at

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He taught formation evaluation at the graduate level in the Petroleum Engineering Department of the University of Southern California for nearly a decade prior to his move from California to Colorado. He and his wife of over 60 years have retired to their home in the countryside outside of the town of Elizabeth, Colorado, USA.

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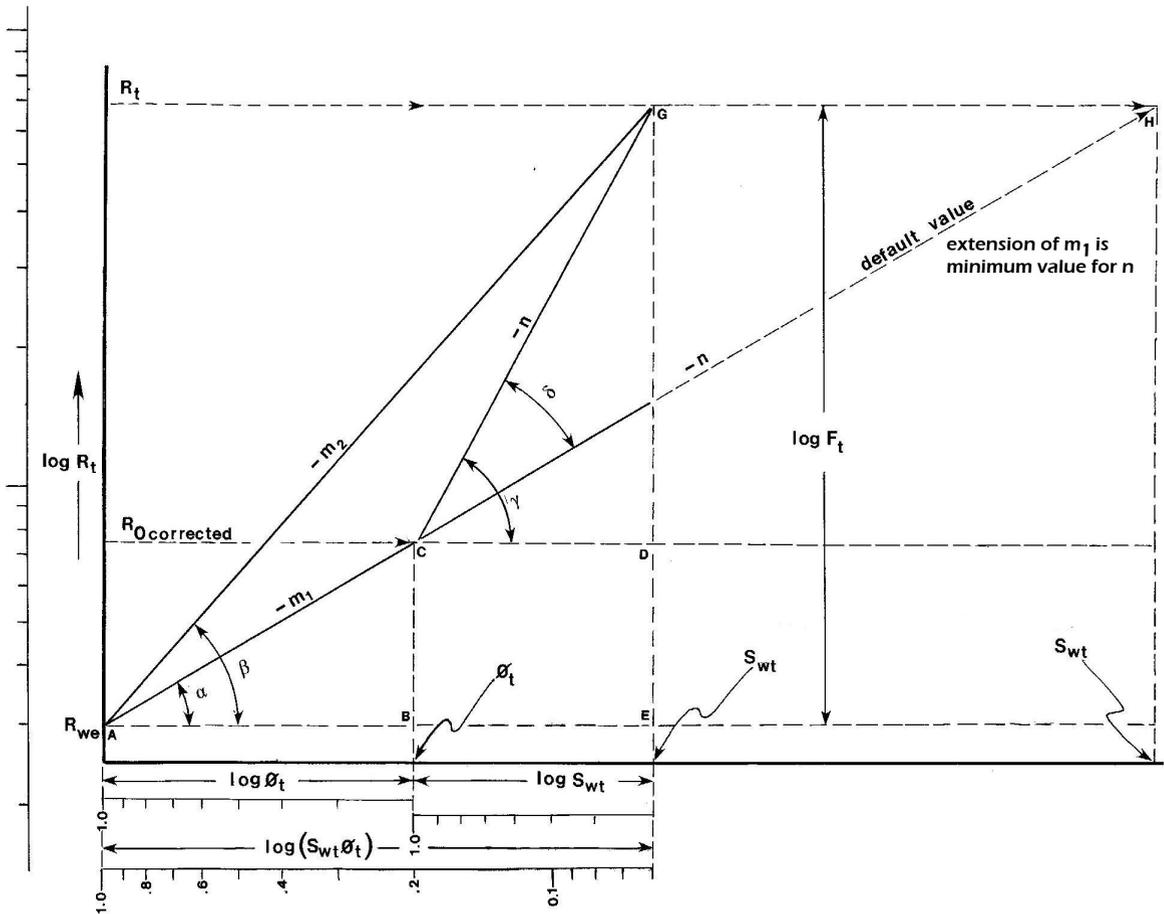


Figure 1. A graphical representation of the model. R_t varies with oil saturation and distribution. Slope CG representing exponent n varies with the electrical interference caused by the presence of oil regardless of saturation value. S_{wt} is estimated by the downward projection of the intersection of the slope CG with resistivity level R_t . See text for full explanation. Based on Ransom (1974,1995). This model can be used with measurements responsive to oil and gas other than resistivity.

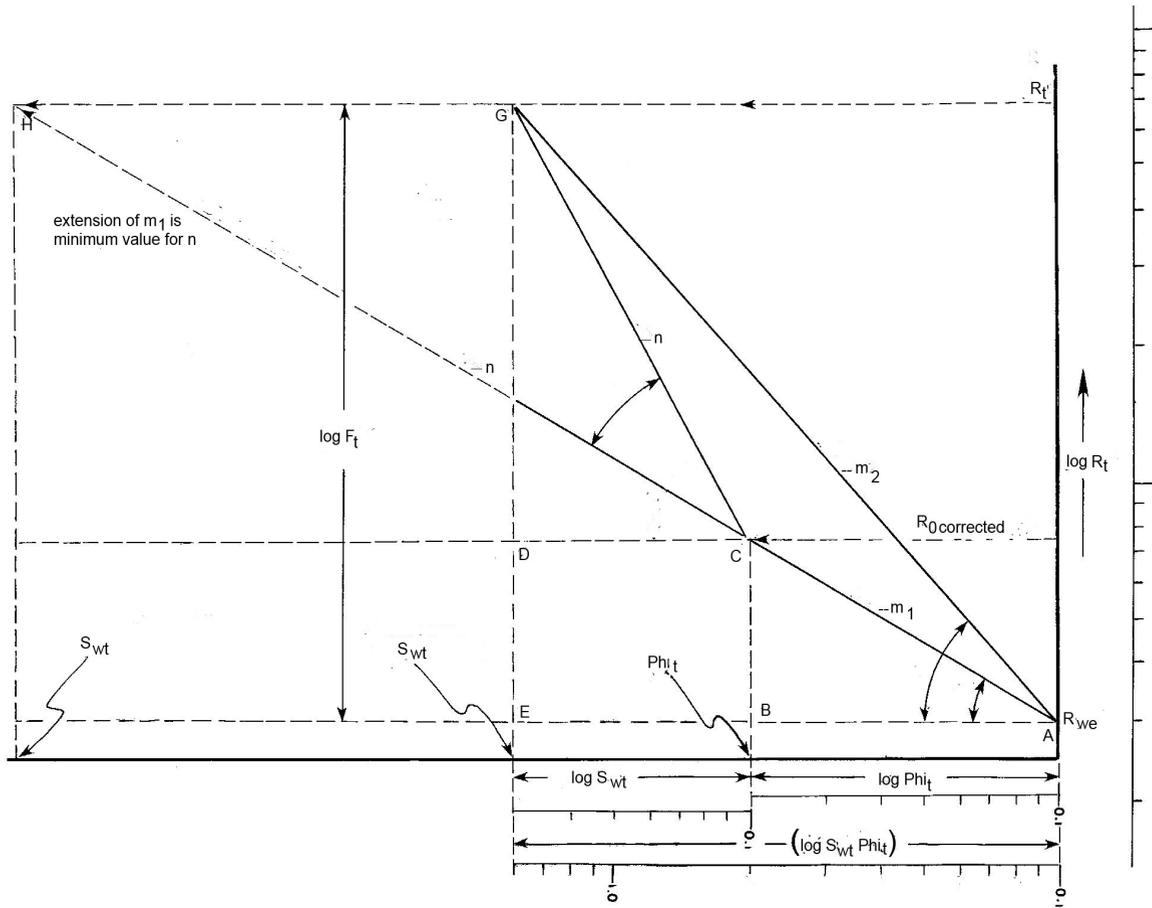


FIGURE 1 Reversed format. Same as Figure 1 above, but reversed for convenience of readers familiar with left-facing format.

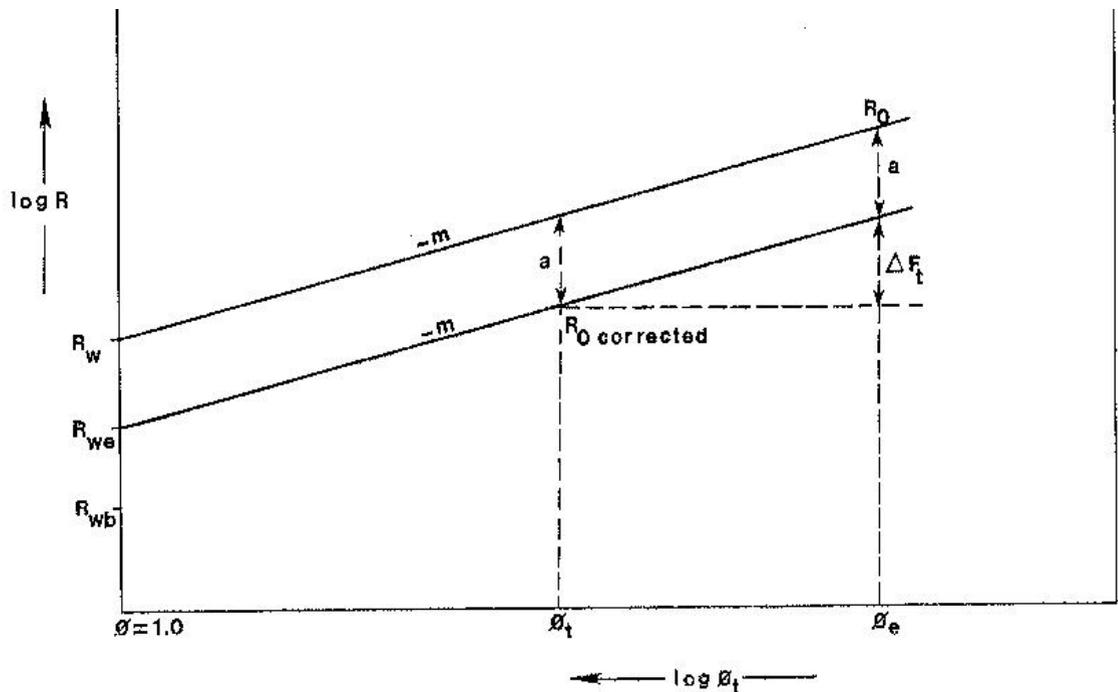


Figure 2. When $S_{wt} = 1.0$. A detailed portion of the graphical model showing how R_{we} results from a mixture of waters R_w and R_{wb} in a shaly sand. The effect of the more conductive pseudo water represented by R_{wb} produces the typical a multiplier of R_w . This figure and its resulting R_{we} corresponds to Eq.(1a) only. Exponent m is an intrinsic property of the rock. Based on Ransom (1974).

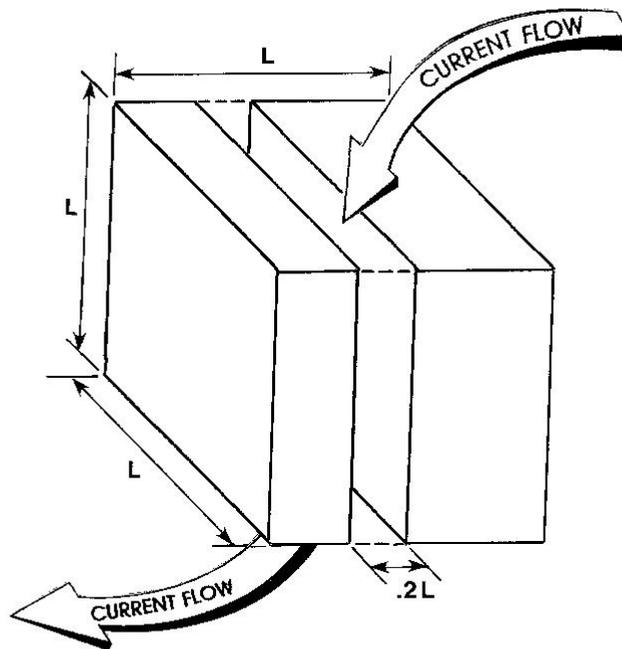


Figure 3. An insulating cube with a 20% void filled with water. Electrical-survey current is flowing through the cube from top to bottom. This cube is a visual aid in the development of the formation factor. See text for discussion. From Ransom (1984, 1995).

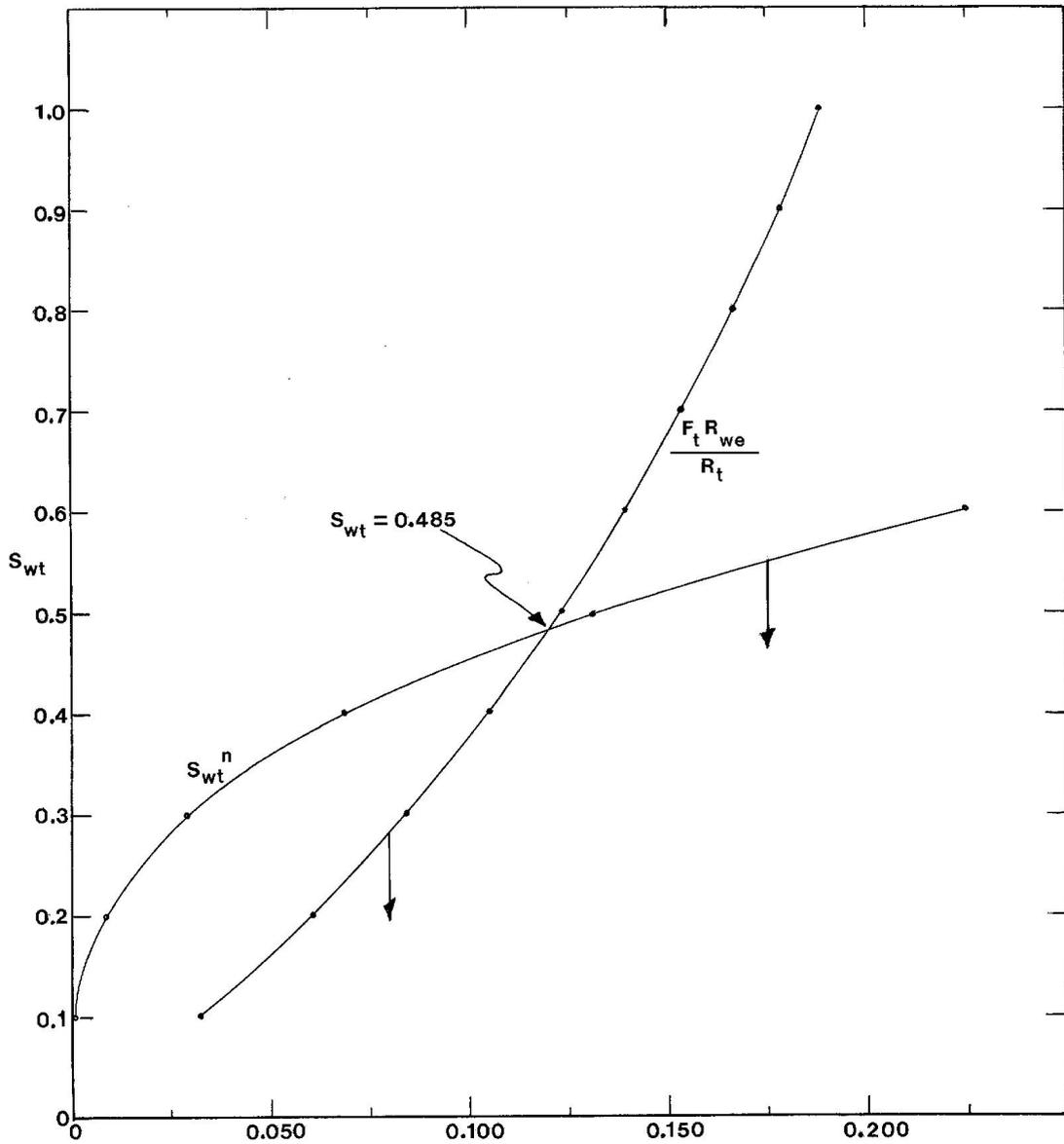


Figure 4. A crossplot illustrating the iteration of S_{wt} for the value of S_{wt} that satisfies both sides of Equation (4b). See the text for discussion.

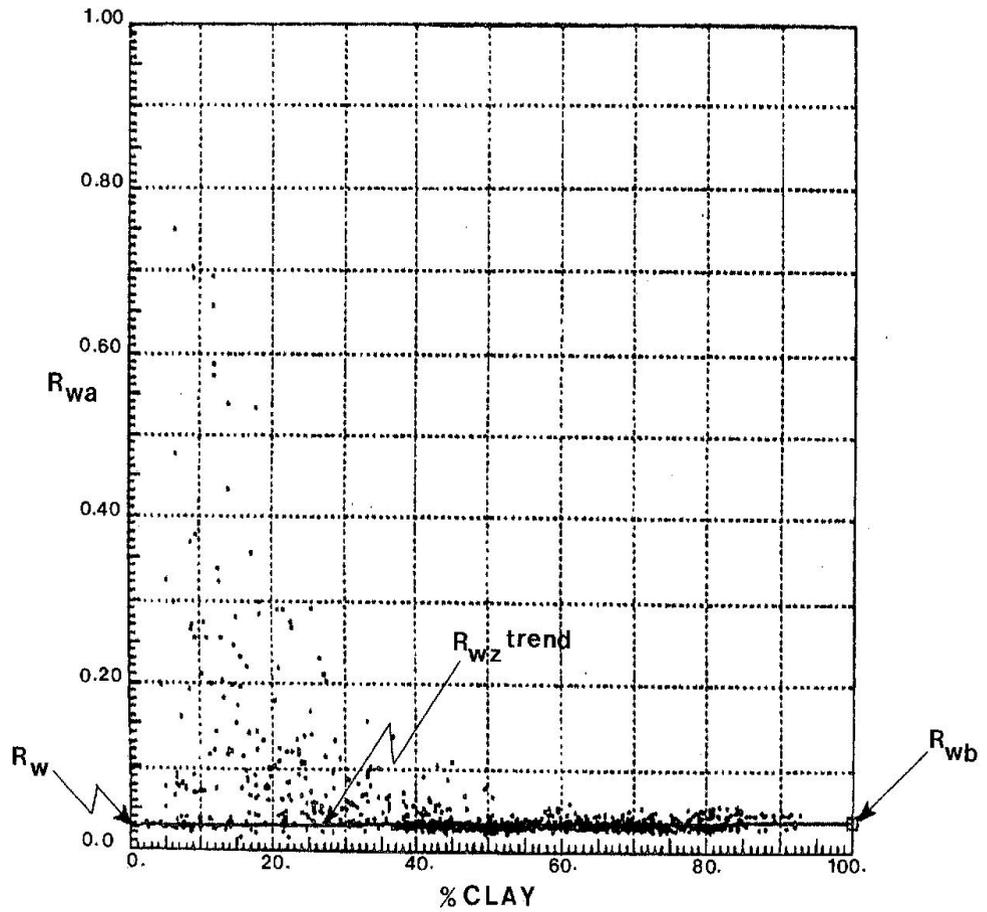


Figure 5. The crossplot of R_{wa} versus *Clayiness* where the identified trend is extrapolated in both directions to evaluate R_w and R_{wb} at 0% and 100% clayiness, respectively. See text for discussion. From Ransom (1995), courtesy of John Wiley & Sons, Inc.

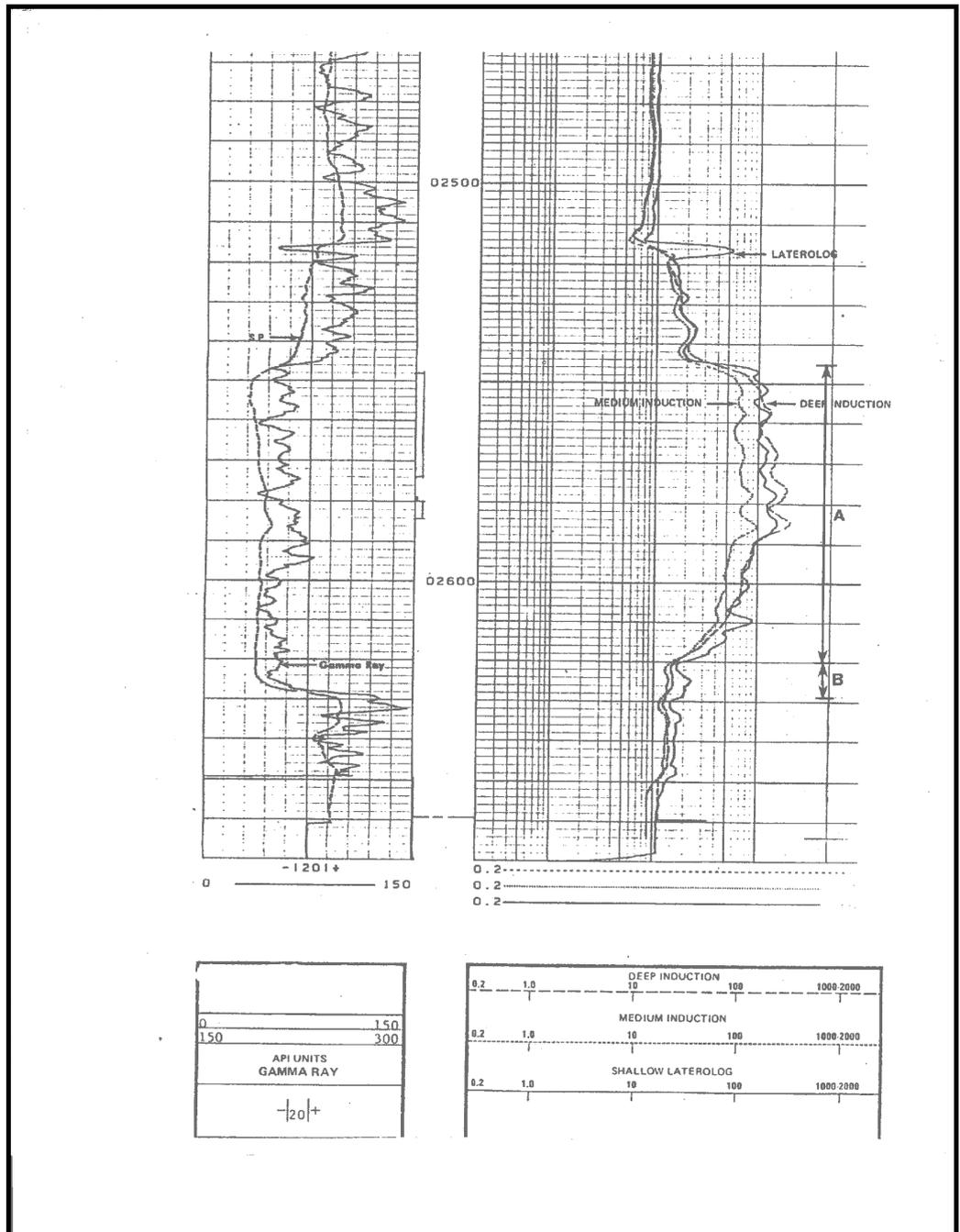


Figure 7. Well 1. The well and reservoir bed in this figure and the following Figure 8. were drilled in the same field. Zone A in this bed was tested over the intervals marked in the depth track and produced much water and little or no oil. No other information is available. Why did Zone A not produce oil? See text for discussion.

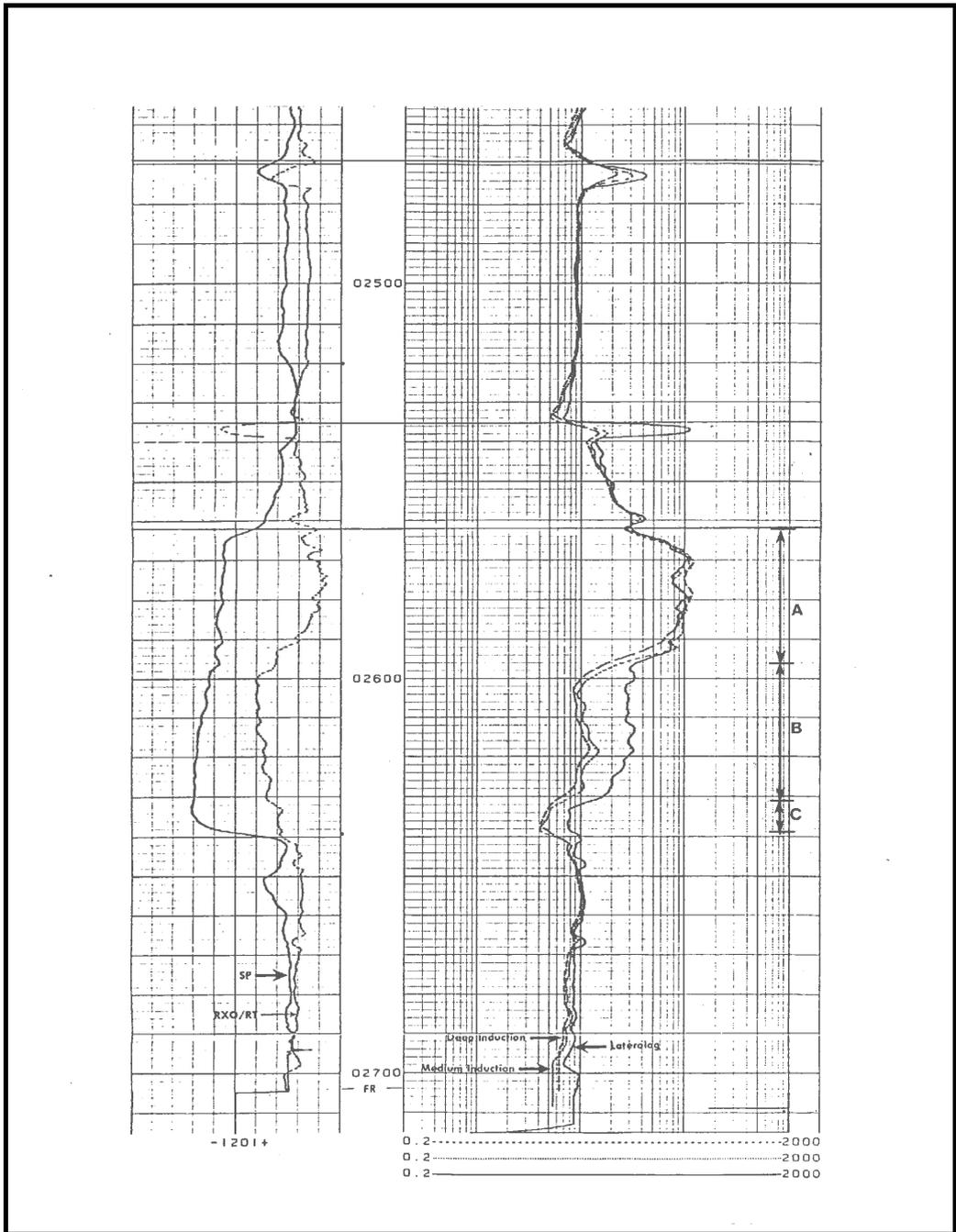


Figure 8. Well 2. The well and reservoir bed in this figure and the preceding Figure 7 were drilled in the same field. No other information is available for this well. Would Zone A produce oil? See text for discussion.

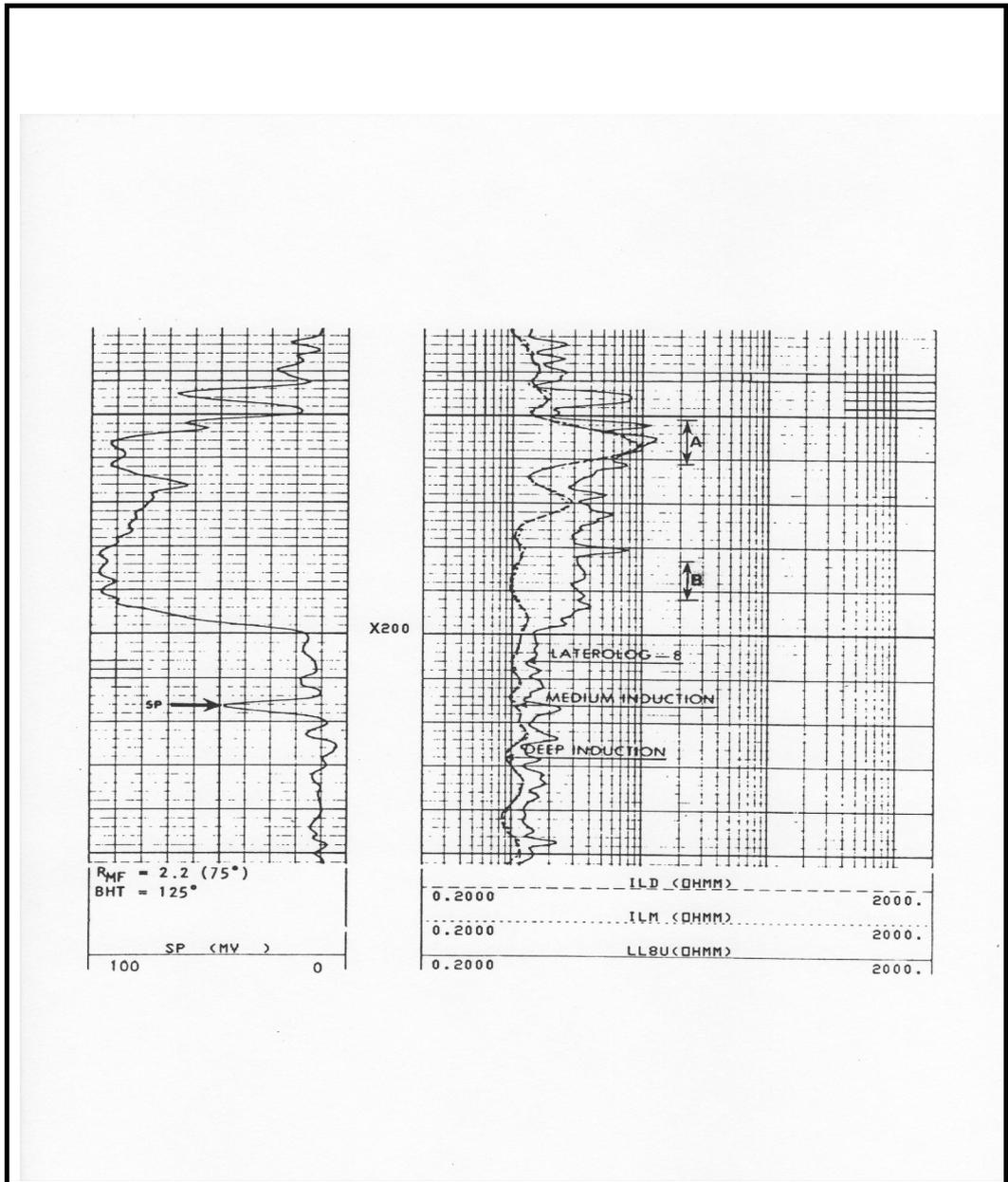


Figure 9. Well 3. An example of a resistivity well log showing the presence of both water and oil in a quality reservoir bed exhibiting considerably lower resistivity than in Wells 1 and 2. Zone A will produce water-free oil. Why? Courtesy of Schlumberger.

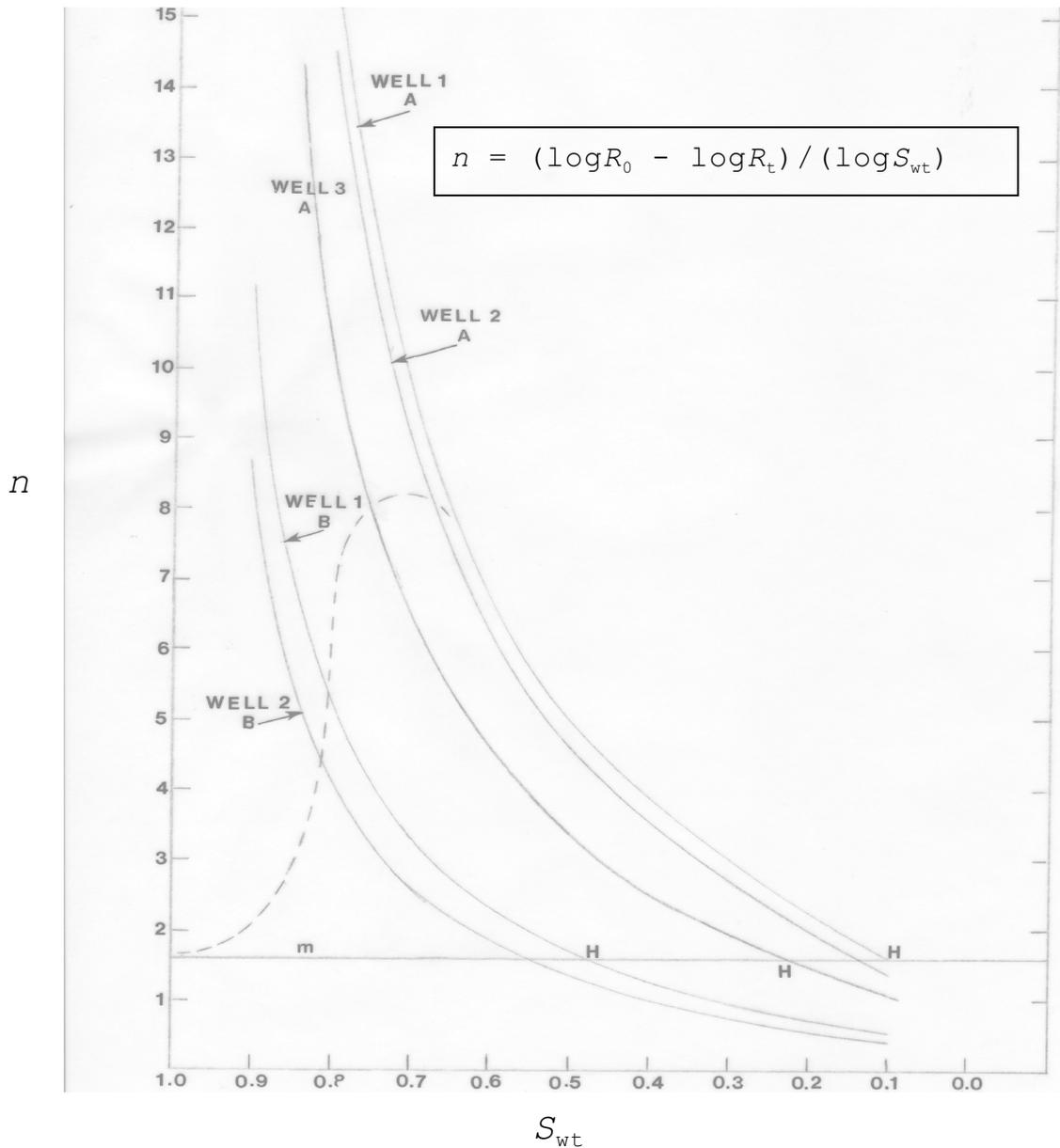


Figure 10. Effectiveness of electrical interference by oil in oil-wet and water-wet rocks. This figure is a chart of the behavior of S_{wt} in response to the calculated values of exponent n (solid lines) from Well 1 (Figure 7), Well 2 (Figure 8), and Well 3 (Figure 9). This behavior is predicted by slope CG of triangle CDG of the model in **Figure 1**. See text for discussion and limitations of n .